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## Chapter - 6: Triangles

### Exercise 6.1 (Page 122 of Grade 10 NCERT)

**Q1.** Fill in the blanks using the correct word given in brackets:

- (i) All circles are \_\_\_\_\_. (congruent, similar)
- (ii) All squares are \_\_\_\_\_. (similar, congruent)
- (iii) All \_\_\_\_\_ triangles are similar. (isosceles, equilateral)
- (iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are \_\_\_\_\_ and (b) their corresponding sides are \_\_\_\_\_. (equal, proportional)

**(i) Reasoning:**

As we know that two similar figures have the same shape but not necessarily the same size. (Same size means radii of the circles are equal)

**Solution:**

Similar. Since the radii of all the circles are not equal.

**(ii) Reasoning:**

As we know that two similar figures have the same shape but not necessarily the same size. (same size means sides of the squares are equal.)

**Solution:**

Similar. Since the sides of the squares are not given equal.

**(iii) Reasoning:**

An equilateral triangle has equal sides and equal angles .

**Solution:**

Equilateral. Each angle in an equilateral triangle is \_\_\_\_\_ .

**(iv) Reasoning:**

As we know that two polygons of same number of sides are similar if their corresponding angles are equal and all the corresponding sides are in the same ratio or proportion.

- i. Since the polygons have same number of sides, we can find each angle using formula,  $\left( \frac{2n-4}{n} \times 90^\circ \right)$ . Here 'n' is number of sides of a polygon.
- ii. We can verify by comparing corresponding sides.

**Solution:**

- (a) Equal
- (b) Proportional



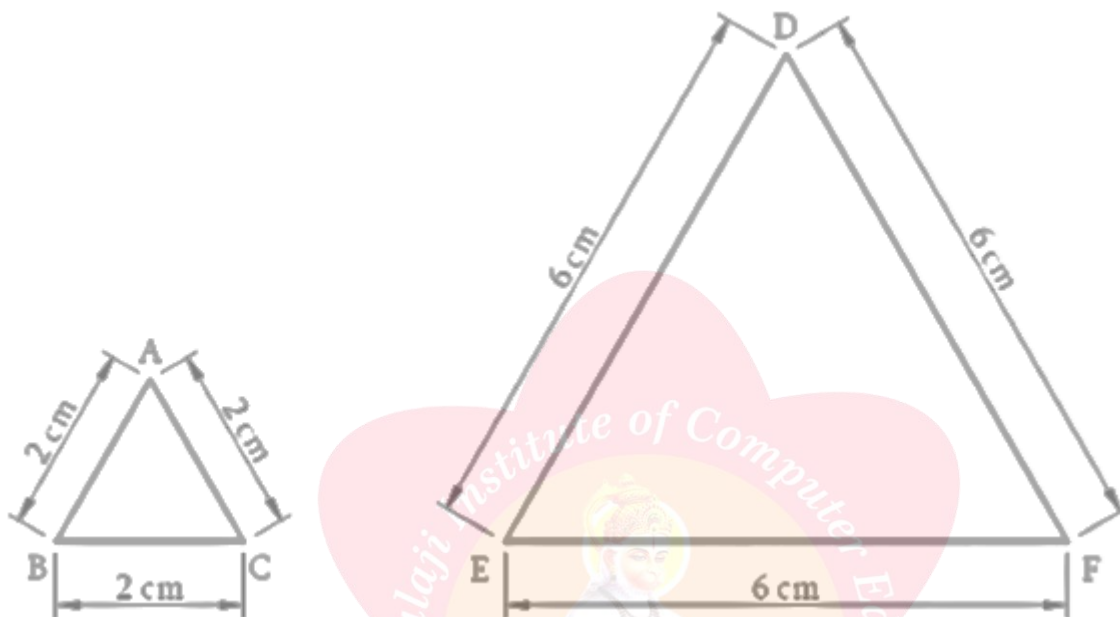
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Q2. Give two different examples of pair of

- (i) similar figures
- (ii) non-similar figures

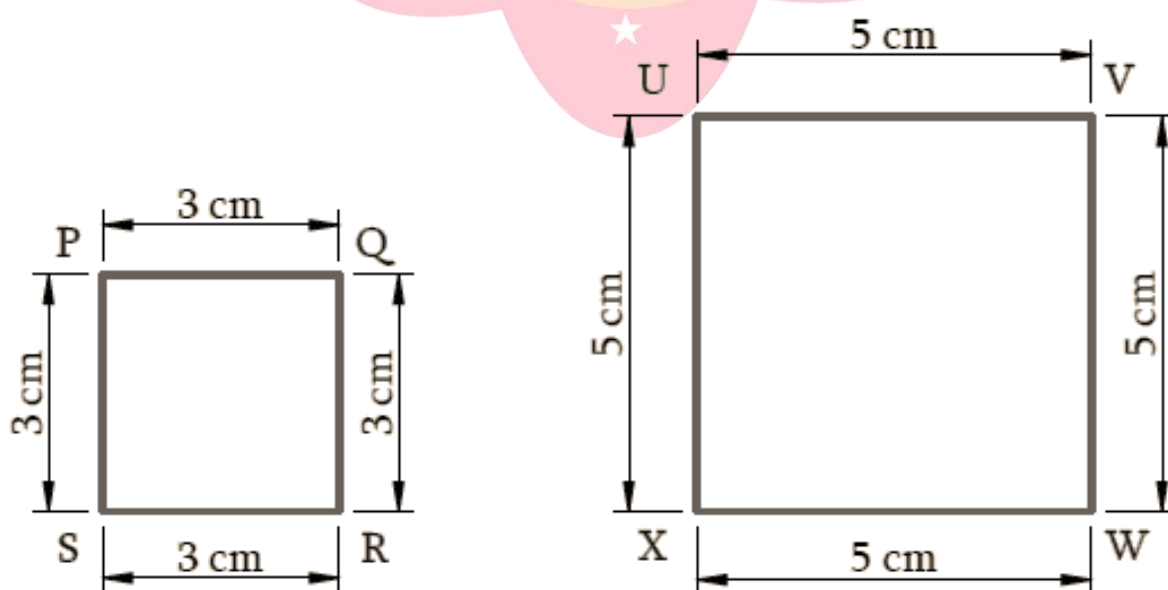
**Solution (i):**

- (i) Two equilateral triangles of sides 2cm and 6cm.



$\triangle ABC \sim \triangle DEF$  (~ is similar to)

- (ii) Two squares of sides 3cm and 5cm.



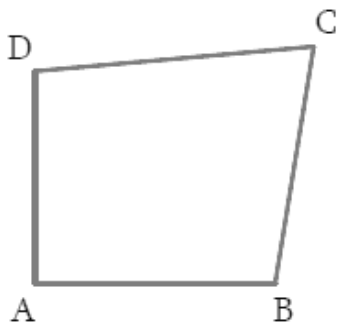
$\square PQRS \sim \square UVWX$  (~ is similar to)



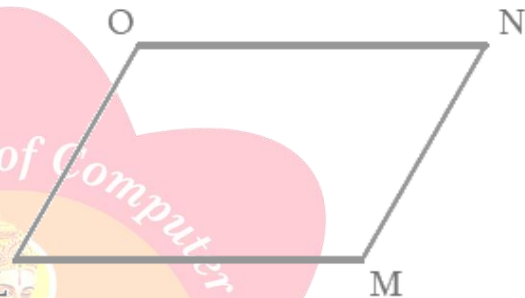
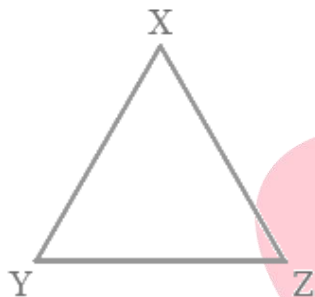
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## Solution (ii):

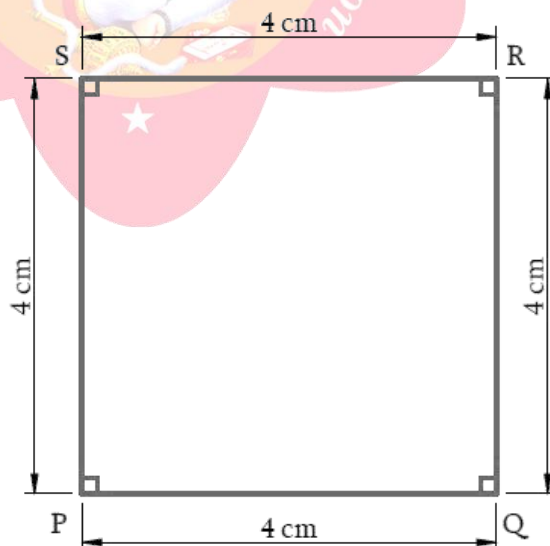
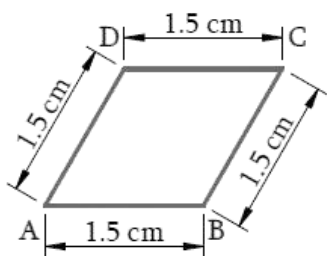
(i) A quadrilateral and a rectangle.



(ii) A triangle and a parallelogram



**Q3.** State whether the following quadrilaterals are similar or not:



## Reasoning:

Two polygons of the same number of sides are similar, if

- (i) all the corresponding angles are equal and
- (ii) all the corresponding sides are in the same ratio (or proportion).



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## Solution:

In Quadrilaterals ABCD and PQRS

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DA}{SP} = \frac{3}{8}$$

⇒ Corresponding sides are in proportion

But  $\angle A \neq \angle P$ ;  $\angle B \neq \angle Q$

⇒ Corresponding angles are not equal

*ABCD*      *PQRS*

No, Quadrilateral ABCD is not similar to Quadrilateral PQRS.

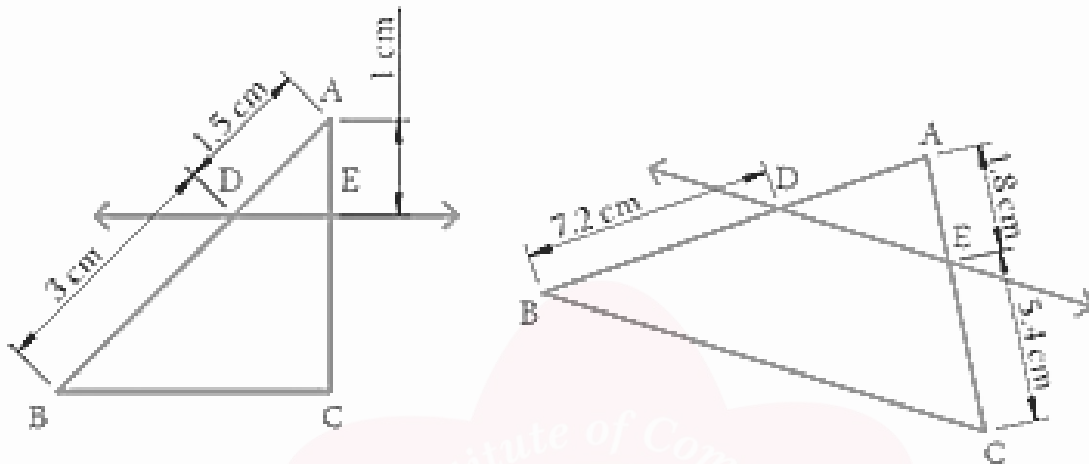




## Chapter - 6: Triangles

### Exercise 6.2 (Page 128 of Grade 10 NCERT)

Q1. In Fig. 6.17, (i) and (ii),  $DE \parallel BC$ . Find EC in (i) and AD in (ii)



#### Reasoning:

As we all know the Basic Proportionality Theorem (B.P.T) or (Thales Theorem)

Two triangles are similar if

- (i) Their corresponding angles are equal
- (ii) Their corresponding sides are in the same ratio (or proportion)

#### Solution:

(i) In,  $\triangle ABC$

$$BC \parallel DE$$

In  $\triangle ABC$  &  $\triangle ADE$

$$\angle ABC = \angle ADE \quad [\text{corresponding angles}]$$

$$\angle ACB = \angle AED \quad [\text{corresponding angles}]$$

$$\angle A = \angle A \text{ common}$$

$$\Rightarrow \triangle ABC \sim \triangle ADE$$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{EC}$$

$$EC = \frac{3 \times 1}{1.5}$$

$$EC = 2$$



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1.5

$EC = 2\text{cm}$





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(ii) Similarly,  $\triangle ABC \sim \triangle ADE$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$AD = \frac{7.2 \times 1.8}{5.4}$$

$$AD = 2.4 \text{ cm}$$

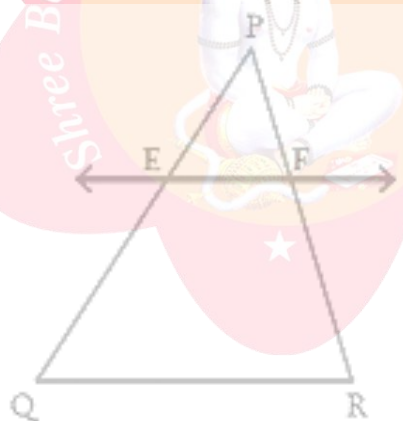
**Q2.** E and F are points on the sides PQ and PR respectively of a  $\triangle PQR$ . For each of the following cases, state whether  $EF \parallel QR$ :

- (i)  $PE = 3.9 \text{ cm}$ ,  $EQ = 3 \text{ cm}$ ,  $PF = 3.6 \text{ cm}$  and  $FR = 2.4 \text{ cm}$
- (ii)  $PE = 4 \text{ cm}$ ,  $QE = 4.5 \text{ cm}$ ,  $PF = 8 \text{ cm}$  and  $RF = 9 \text{ cm}$
- (iii)  $PQ = 1.28 \text{ cm}$ ,  $PR = 2.56 \text{ cm}$ ,  $PE = 0.18 \text{ cm}$  and  $PF = 0.36 \text{ cm}$

**(i) Reasoning:**

As we know that a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side (converse of BPT)

**Solution:**



Here,

$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

and

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Hence,

$$\frac{PE}{EQ} \neq \frac{PF}{FR}$$

According to converse of BPT, EF is not parallel to QR.



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## (ii) Reasoning:

As we know that a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side (converse of BPT)

### Solution:

Here,

$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

and

$$\frac{PF}{FR} = \frac{8}{9}$$

Hence,

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

According to converse of BPT,  $EF \parallel QR$

## (iii) Reasoning:

As we know that a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side (converse of BPT)

### Solution:

Here,

$$PQ = 1.28\text{cm and } PE = 0.18\text{cm}$$

$$\begin{aligned} EQ &= PQ - PE \\ &= (1.28 - 0.18)\text{cm} \\ &= 1.10\text{cm} \end{aligned}$$

$$PR = 2.56\text{cm and } PF = 0.36\text{cm}$$

$$\begin{aligned} FR &= PR - PF \\ &= (2.56 - 0.36)\text{cm} \\ &= 2.20\text{cm} \end{aligned}$$

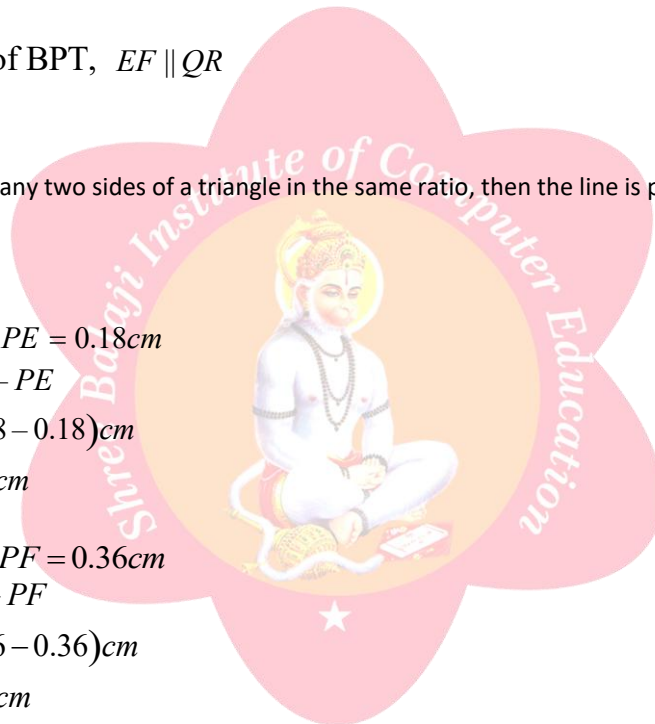
Now,

$$\frac{PE}{EQ} = \frac{0.18\text{cm}}{1.10\text{cm}} = \frac{18}{110} = \frac{9}{55}$$

$$\frac{PF}{FR} = \frac{0.36\text{cm}}{2.20\text{cm}} = \frac{36}{220} = \frac{9}{55}$$

$$\Rightarrow \frac{PE}{EQ} = \frac{PF}{FR}$$

According to converse of BPT,  $EF \parallel QR$

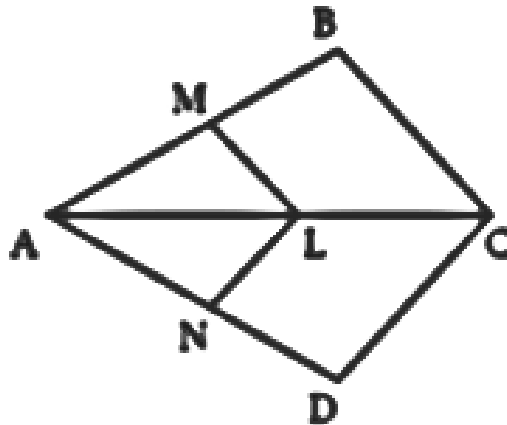




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**Q3.** In Fig. 6.18, if  $LM \parallel CB$  and  $LN \parallel CD$ , prove that

$$\frac{AM}{AB} = \frac{AN}{AD}$$



## Reasoning:

As we know if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

## Solution:

In  $\triangle ABC$

$$\begin{aligned} LM &\parallel CB \\ \frac{AM}{MB} &= \frac{AL}{LC} \dots\dots\dots (\text{Eq 1}) \end{aligned}$$

In  $\triangle ACD$

$$\begin{aligned} LN &\parallel CD \\ \frac{AN}{DN} &= \frac{AL}{LC} \dots\dots\dots (\text{Eq 2}) \end{aligned}$$

From equations (1) and (2)

$$\frac{AM}{MB} = \frac{AN}{DN}$$

$$\Rightarrow \frac{MB}{AM} = \frac{DN}{AN}$$

Adding 1 on both sides

$$\frac{MB}{AM} + 1 = \frac{DN}{AN} + 1$$

$$\frac{MB + AM}{AM} = \frac{DN + AN}{AN}$$

$$\frac{AB}{AM} = \frac{AD}{AN}$$

$$\frac{AB}{AM} = \frac{AD}{AN}$$

$$\frac{AM}{AB} = \frac{AN}{AD}$$

$$\frac{AM}{AB} = \frac{AN}{AD}$$

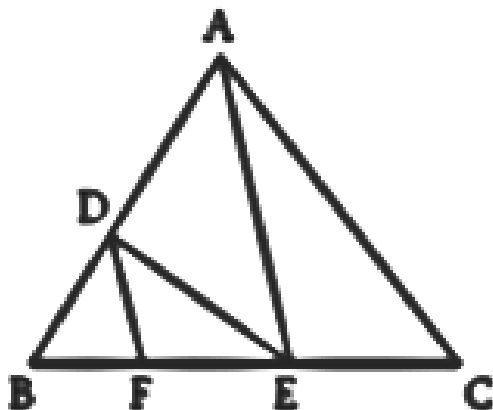
$$\frac{AM}{AB} = \frac{AN}{AD}$$



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**Q4.** In Fig. 6.19,  $DE \parallel AC$  and  $DF \parallel AE$ . Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}$$



## Reasoning:

As we know if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

## Solution:

In  $\triangle ABC$

$$DE \parallel AC$$

$$\frac{BD}{AD} = \frac{BE}{EC} \dots\dots\dots (i)$$

In  $\triangle ABE$

$$DF \parallel AE$$

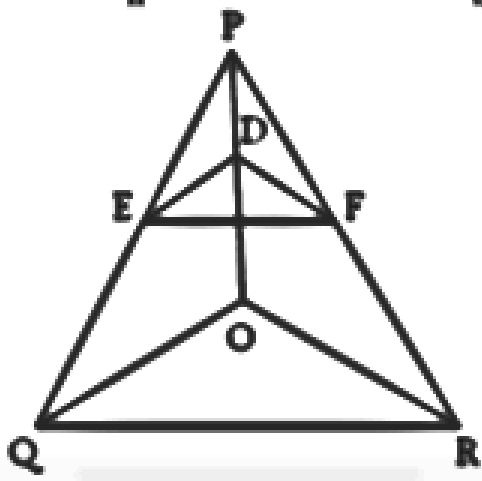
$$\frac{BD}{AD} = \frac{BF}{FE} \dots\dots\dots (ii)$$

From (i) and (ii)

$$\frac{BD}{AD} = \frac{BE}{EC} = \frac{BF}{FE}$$

$$\frac{BE}{EC} = \frac{BF}{FE}$$

**Q5.** In Fig. 6.20,  $DE \parallel OQ$  and  $DF \parallel OR$ . Show that  $EF \parallel QR$ .





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## Reasoning:

As we know If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

## Solution:

In  $\triangle POQ$

$$DE \parallel OQ(\text{given})$$

$$\frac{PE}{EQ} = \frac{PD}{DO} \dots\dots\dots (1)$$

In  $\triangle POR$

$$DF \parallel OR(\text{given})$$

$$\frac{PF}{FR} = \frac{PD}{DO} \dots\dots\dots (2)$$

From (1) & (2)

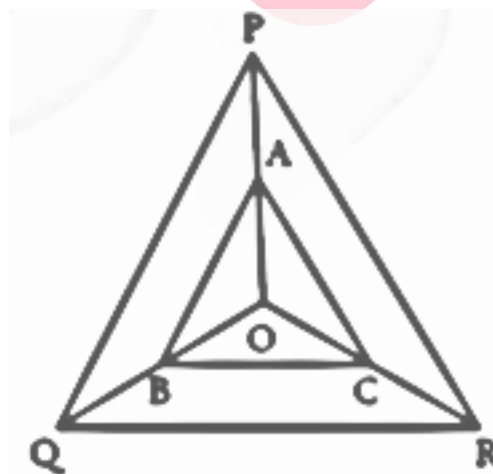
$$\frac{PE}{EQ} = \frac{PF}{FR} = \frac{PD}{DO}$$
$$\frac{PE}{EQ} = \frac{PF}{FR}$$

In  $\triangle PQR$

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\therefore QR \parallel EF$  (Converse of BPT)

**Q6.** In Fig. 6.21, A, B and C are points on OP, OQ and OR respectively such that  $AB \parallel PQ$  and  $AC \parallel PR$ . Show that  $BC \parallel QR$ .



## Reasoning:

As we know If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.



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**Solution:**

In  $\triangle OPQ$

$$AB \parallel PQ \text{ (given)}$$

$$\frac{OA}{AP} = \frac{OB}{BQ} \dots\dots\dots (i)$$

[ Thales Theorem(BPT)]

In  $\triangle OPR$

$$AC \parallel PQ \text{ (given)}$$

$$\frac{OA}{AP} = \frac{OC}{CR} \dots\dots\dots (ii)$$

[ Thales Theorem(BPT)]

From (i) & (ii)

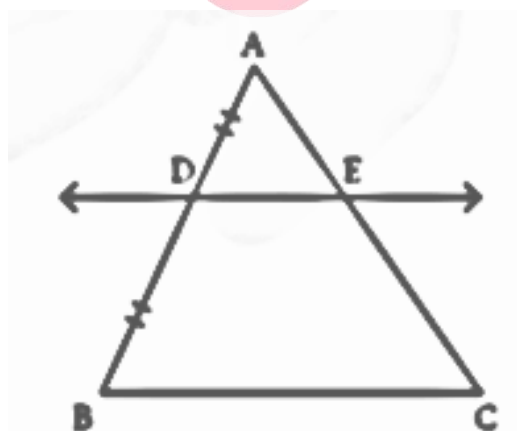
$$\frac{OA}{AP} = \frac{OB}{BQ} = \frac{OC}{CR}$$
$$\frac{OB}{BQ} = \frac{OC}{CR}$$

Now, In  $\triangle OQR$

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

$BC \parallel QR$  [ Converse of BPT]

**Q7.** Using Theorem 6.1, prove that a line drawn through the midpoint of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).



**Reasoning:**

We know that theorem 6.1 states that “If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio (BPT)”.



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## Solution:

In  $\triangle ABC$ , D is the midpoint of AB

Therefore,

$$AD = BD$$

$$\frac{AD}{BD} = 1$$

Now,

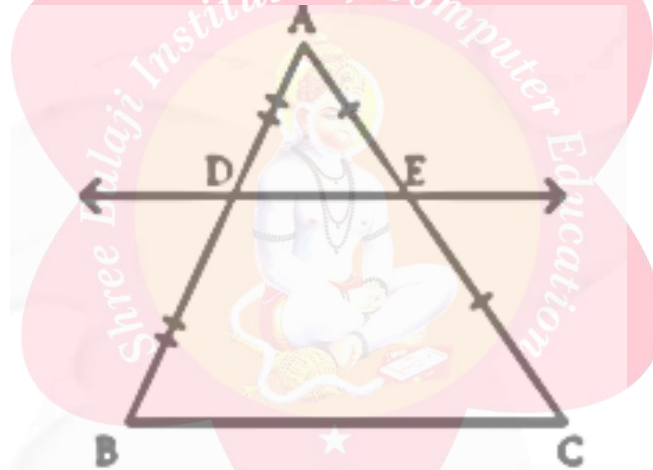
$$DE \parallel BC$$

$$\Rightarrow \frac{AE}{EC} = \frac{AD}{BD} \quad [\text{Theorem 6.1}]$$

$$\Rightarrow \frac{AE}{EC} = 1$$

Hence, E is the midpoint of AC.

**Q8.** Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).



## Reasoning:

We know that theorem 6.2 tells us if a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. (Converse of BPT)

## Solution:

In  $\triangle ABC$

D is the midpoint of AB

$$AD = BD$$

$$\frac{AD}{BD} = 1 \dots\dots\dots(i)$$

E is the midpoint of AC

$$AE = CE$$

$$\frac{AE}{CE} = 1 \dots\dots\dots(ii)$$



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From (i) and (ii)

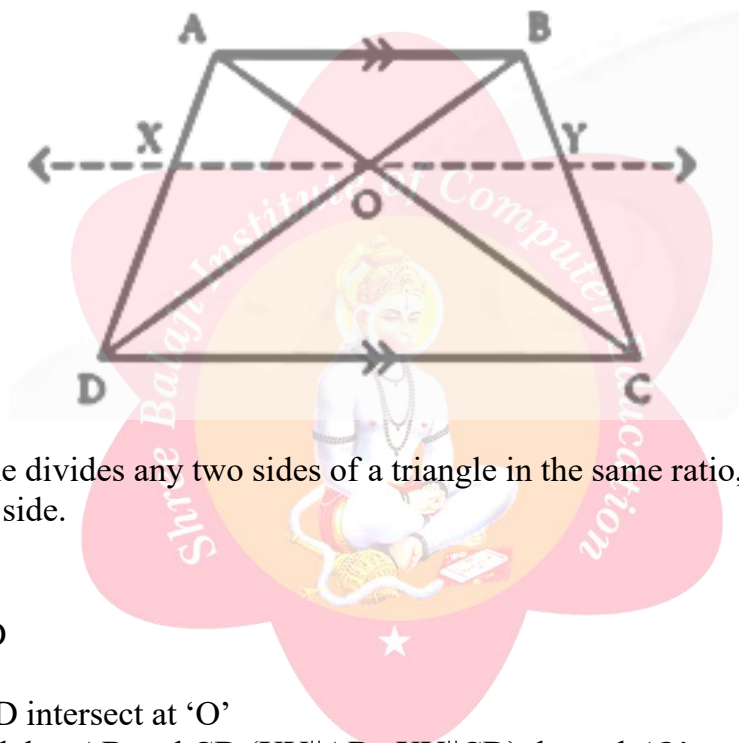
$$\frac{AD}{BD} = \frac{AE}{BE} = 1$$

$$\frac{AD}{BD} = \frac{AE}{BE}$$

According to theorem 6.2 (Converse of BPT),

$$DE \parallel BC$$

**Q9.** ABCD is a trapezium in which  $AB \parallel DC$  and its diagonals intersect each other at the point O. Show that  $\frac{AO}{BO} = \frac{CO}{DO}$



## Reasoning:

As we know If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

## Solution:

In trapezium ABCD

$AB \parallel CD$ , AC and BD intersect at 'O'

Construct XY parallel to AB and CD ( $XY \parallel AB$ ,  $XY \parallel CD$ ) through 'O'

In  $\triangle ABC$

$$OY \parallel AB \text{ (construction)}$$

According to theorem 6.1 (BPT)

$$\frac{BY}{CY} = \frac{AO}{OC} \dots\dots\dots (I)$$

In  $\triangle BCD$

$$OY \parallel CD (\because \text{construction})$$

According to theorem 6.1 (BPT)

$$\frac{BY}{CY} = \frac{OB}{OD} \dots\dots\dots (II)$$



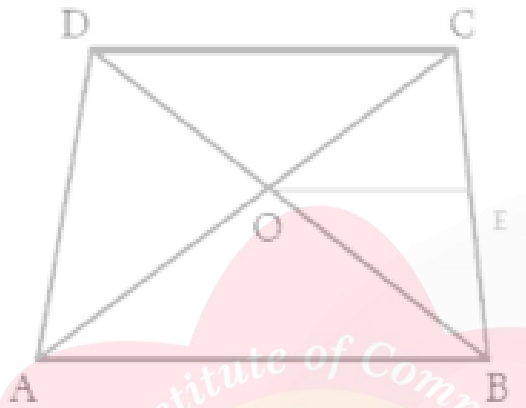
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From (I) and (II)

$$\begin{aligned} OA &= OB \\ OC &= OD \\ \Rightarrow \frac{OA}{OB} &= \frac{OC}{OD} \end{aligned}$$

**Q10.** The diagonals of a quadrilateral ABCD intersect each other at the point 'O'

such that  $\frac{AO}{BO} = \frac{CO}{DO}$ . Show that ABCD is a trapezium.



## Reasoning:

As we know If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

## Solution:

In quadrilateral ABCD  
Diagonals AC, BD intersect at 'O'

Draw  $OE \parallel AB$

In  $\triangle ABC$

$$OE \parallel AB$$

$$\Rightarrow \frac{OA}{OC} = \frac{BE}{CE} \text{ (BPT) } \dots\dots\dots (1)$$

$$\text{But } \frac{OA}{OB} = \frac{OC}{OD} \text{ (given)}$$

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD} \dots\dots\dots (2)$$

From (1) and (2)

$$\frac{OB}{OD} = \frac{BE}{CE}$$



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InMCD

*OB BE*

*OD CE*

*OEIICD*

*OEIAB* and *OEIICD*

***ABIICD***

*ABCD* is a trapezium



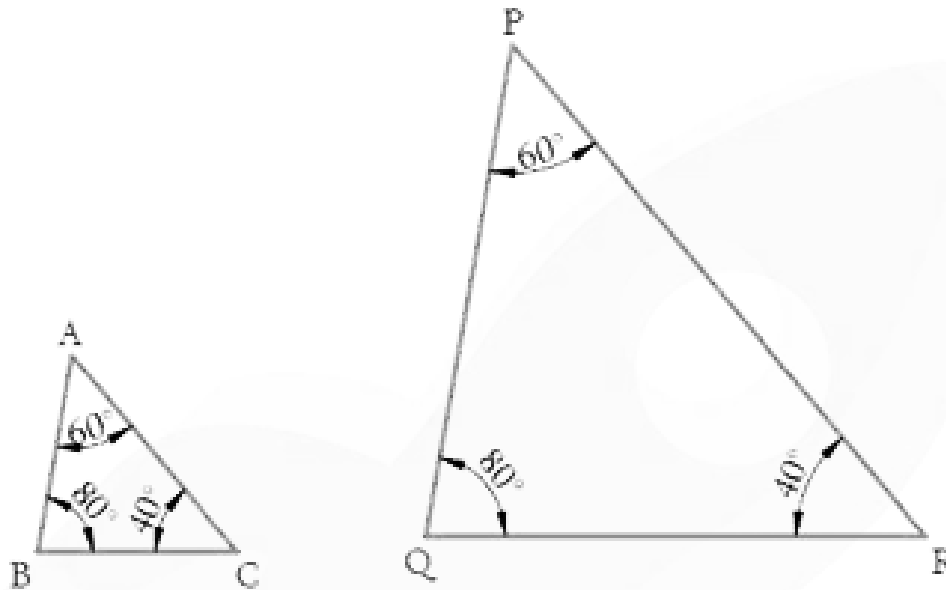


## Chapter - 6: Triangles

### Exercise 6.3(Page 138)

**Q1.** State which pairs of triangles in Fig. 6.34 are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

1)



**Difficulty Level:**

Medium

**Reasoning:**

As we know If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar

This is referred as AAA (Angle – Angle – Angle) criterion of similarity of two triangles.

**Solution:**

In  $\triangle ABC$  and  $\triangle PQR$

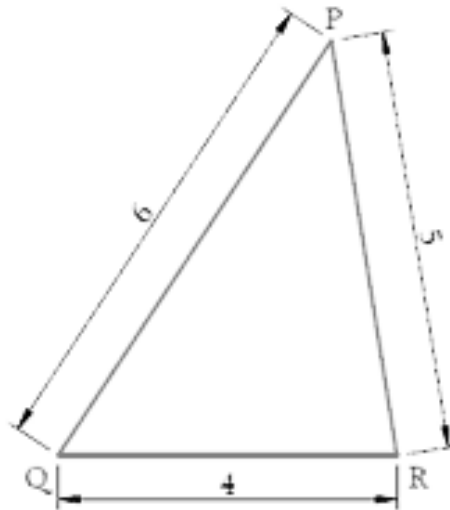
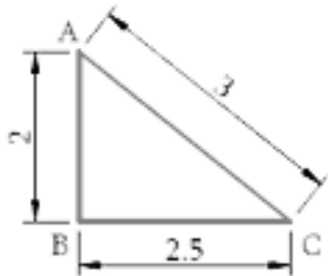
$$\angle A = \angle P = 60$$

$$\angle B = \angle Q = 80$$

$$\angle C = \angle R = 40$$

All the corresponding angles of the triangles are equal.

By AAA criterion  $\triangle ABC \sim \triangle PQR$



## Reasoning:

As we know if in two triangles side of one triangle are proportional to (i.e., in the same ratio of) the side of other triangle, then their corresponding angles are equal and hence the two triangles are similar.

This is referred as SSS (Side – Side – Side) similarity criterion for two triangles.

## Solution:

In  $\triangle ABC$  and  $\triangle QRP$

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{BC}{PR} = \frac{2.5}{5} = \frac{1}{2}$$

$$\frac{AC}{PQ} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{AB}{QR} = \frac{BC}{PR} = \frac{AC}{PQ} = \frac{1}{2}$$

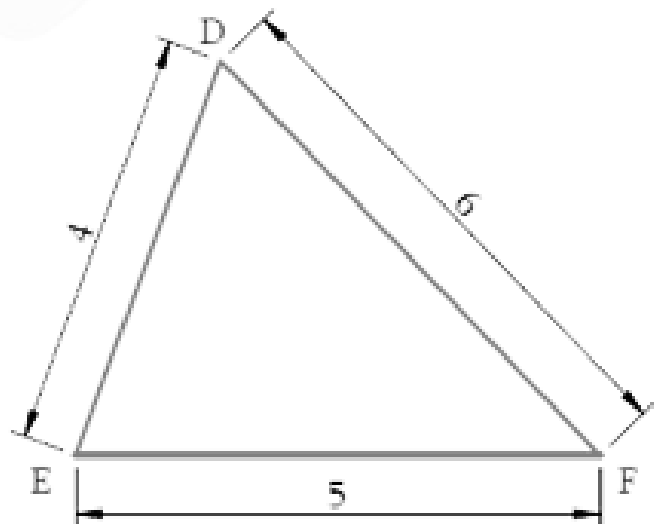
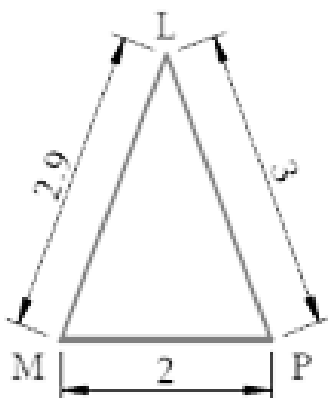
$$\Rightarrow \frac{AB}{QR} = \frac{BC}{PR} = \frac{AC}{PQ} = \frac{1}{2}$$

$$\Rightarrow \frac{AB}{QR} = \frac{BC}{PR} = \frac{AC}{PQ} = \frac{1}{2}$$

All the corresponding sides of two triangles are in same proportion.

By SSS criterion  $\triangle ABC \sim \triangle QPR$

## 3)





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## Reasoning:

As we know if in two triangles side of one triangle are proportional to (i.e., in the same ratio of) the side of other triangle, then their corresponding angles are equal and hence the two triangles are similar.

This is referred as SSS (Side – Side – Side) similarity criterion for two triangles.

## Solution:

In  $\triangle LMP$  and  $\triangle FED$

$$\frac{LM}{FE} = \frac{2.7}{5}$$

$$\frac{MP}{ED} = \frac{2}{4} = \frac{1}{2}$$

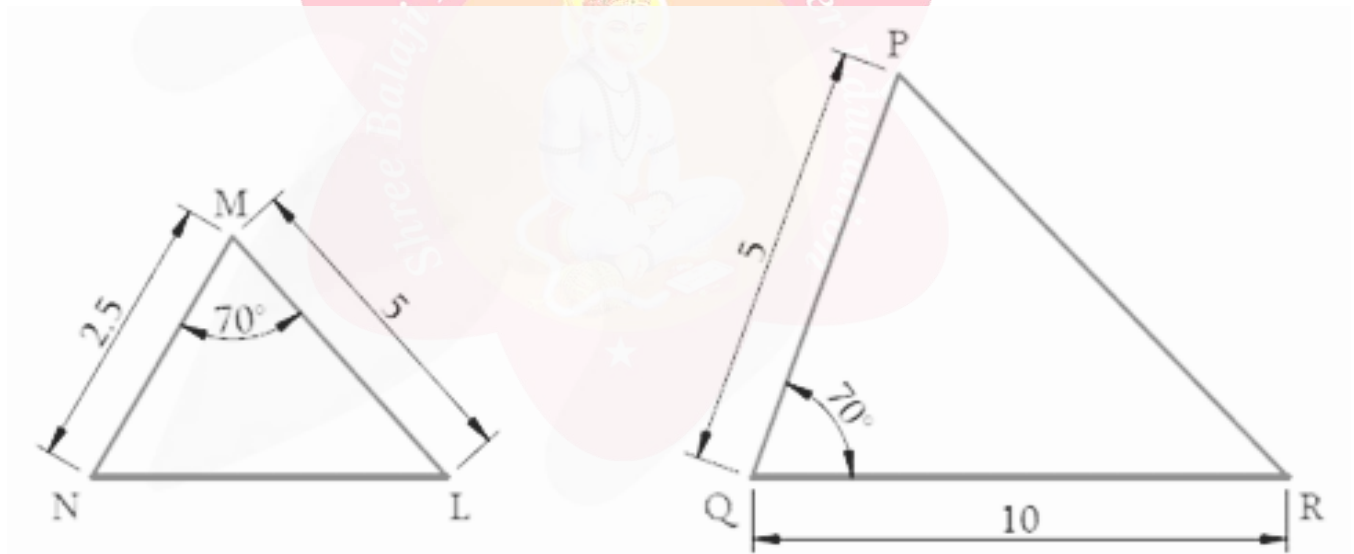
$$\frac{LP}{FD} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{LM}{FE} \neq \frac{MP}{ED} = \frac{LP}{FD}$$

$$\Rightarrow \frac{LM}{FE} \neq \frac{MP}{ED} = \frac{LP}{FD}$$

All the corresponding sides of two triangles are not in same proportion.  
Hence triangles are not similar.  $\triangle LMP$   $\triangle FED$

4)



## Reasoning:

As we know if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

This is referred as SAS (Side – Angle – Side) similarity criterion for two triangles.



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**Solution:**

In  $\triangle NML$  and  $\triangle PQR$

$$\frac{NM}{PQ} = \frac{2.5}{5} = \frac{1}{2}$$

$$\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$$

$$\frac{NM}{PQ} = \frac{ML}{QR} = \frac{1}{2}$$

$$\angle M = \angle Q = 70^\circ$$

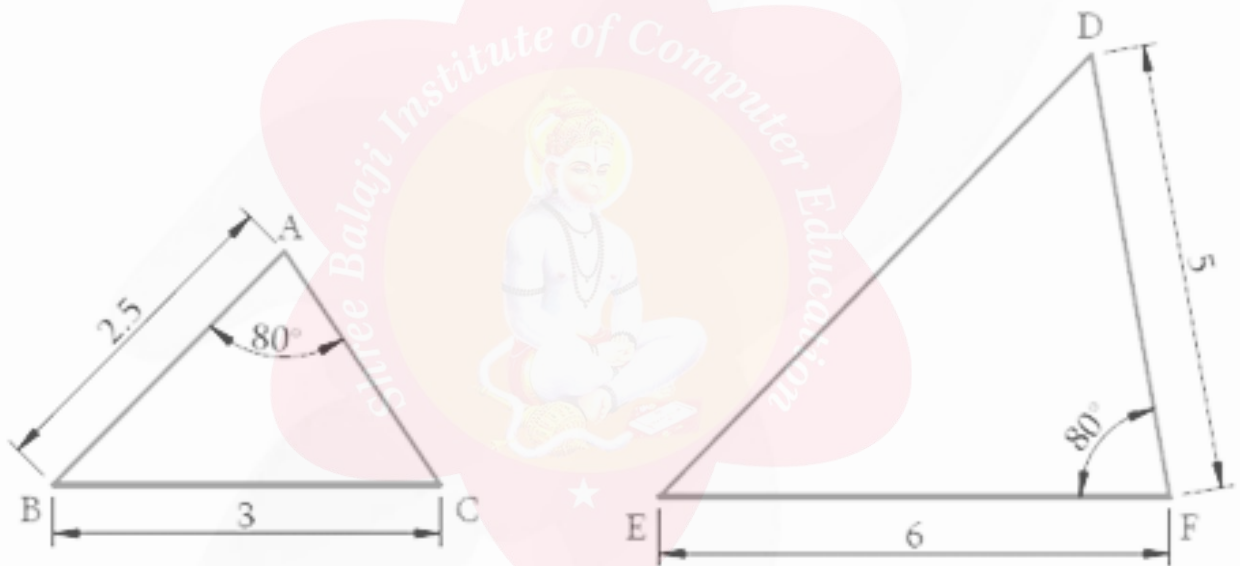
$$\Rightarrow \frac{NM}{PQ} = \frac{ML}{QR} = \frac{1}{2}$$

$$\angle M = \angle Q = 70^\circ$$

One angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional.

By SAS criterion  $\Rightarrow \triangle NML \sim \triangle PQR$

5)



## Reasoning:

As we know if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

This is referred as SAS (Side – Angle – Side) similarity criterion for two triangles.

**Solution:**

In  $\triangle ABC$  and  $\triangle DFE$

$$\frac{AB}{DE} = \frac{2.5}{5} = \frac{1}{2}$$

$$\frac{BC}{EF} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{1}{2}$$

$$\angle B = \angle E = 80^\circ$$



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$$\frac{AB}{DF} = \frac{BC}{EF} = \frac{1}{2}$$





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$$\angle A = \angle F = 80^\circ$$

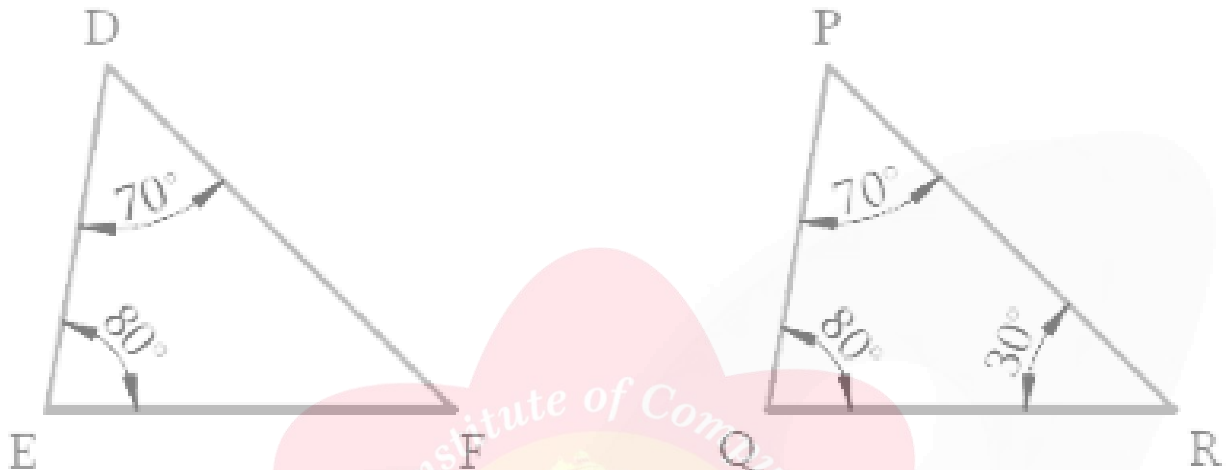
But  $\angle B$  must be equal to  $80^\circ$

( The sides AB, BC includes  $\angle B$ , not  $\angle A$ )

Therefore, SAS criterion is not satisfied

Hence, the triangles are not similar,  $\triangle ABC \not\sim \triangle DFE$

6)



## Reasoning:

As we know If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar

This is referred as AAA (Angle – Angle – Angle) criterion of similarity of two triangles.

## Solution:

In  $\triangle DEF$

$$\angle D = 70^\circ, \angle E = 80^\circ$$

$$\Rightarrow \angle F = 30^\circ \left[ \text{Sum of the angles in a triangle is } 180^\circ \right]$$

Similarly,

In  $\triangle PQR$

$$\angle Q = 80^\circ, \angle R = 30^\circ$$

$$\Rightarrow \angle P = 70^\circ$$

In  $\triangle DEF$  and  $\triangle PQR$

$$\angle D = \angle P = 70^\circ$$

$$\angle E = \angle Q = 80^\circ$$

$$\angle F = \angle R = 30^\circ$$

All the corresponding angles of the triangles are equal.

By AAA criterion  $\triangle DEF \sim \triangle PQR$



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## Alternate method:

### Reasoning:

As we are aware if two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

### Solution:

In  $\triangle DEF$

$$\angle D = 70^\circ, \angle E = 80^\circ$$

$$\Rightarrow \angle F = 30^\circ \quad \left[ \text{Sum of the angles in a triangle is } 180^\circ \right]$$

Now,

In  $\triangle DEF$  and  $\triangle PQR$

$$\angle E = \angle Q = 80^\circ$$

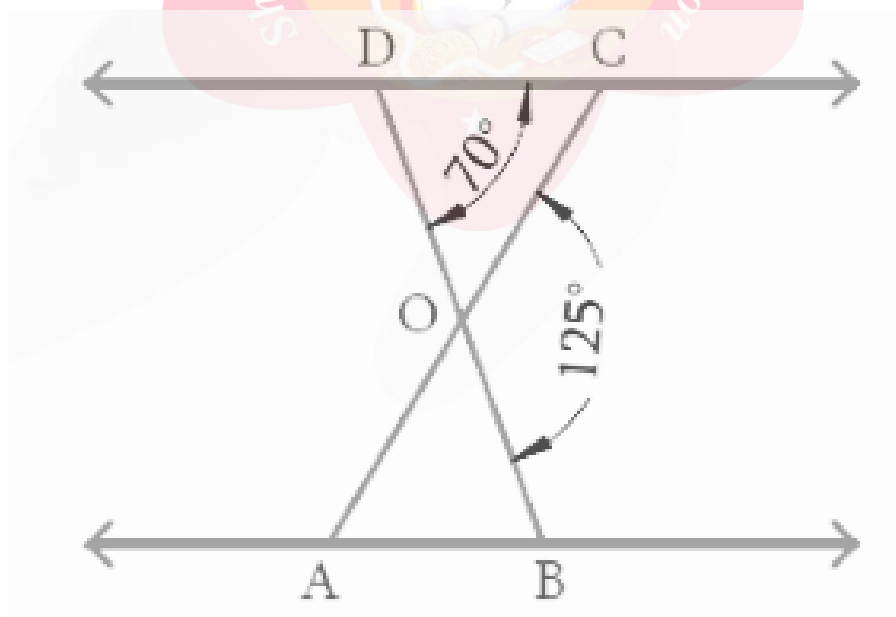
$$\angle F = \angle R = 30^\circ$$

Pair of corresponding angles of the triangles are equal.

By AA criterion  $\triangle DEF \sim \triangle PQR$

**Q2.** In Figure 6.35  $\triangle ODC \sim \triangle OBA$ ,  $\angle BOC = 125^\circ$  and  $\angle CDO = 70^\circ$ . Find  $\angle DOC$ ,  $\angle DCO$  and  $\angle OAB$ .

### Diagram



### Reasoning:

As we are aware if two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.



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## Solution:

In the given figure.

$$\angle DOC = 180 - \angle COB \quad [ \angle DOC \text{ and } \angle COB \text{ from a linear pair} ]$$

$$\angle DOC = 180 - 125$$

$$\angle DOC = 55$$

In  $\triangle ODC$

$$\angle DCO = 180 - (\angle DOC + \angle ODC) \quad [ \text{angle sum property} ]$$

$$\angle DCO = 180 - (55 + 70)$$

$$\angle DCO = 55$$

In  $\triangle ODC$  and  $\triangle OBA$

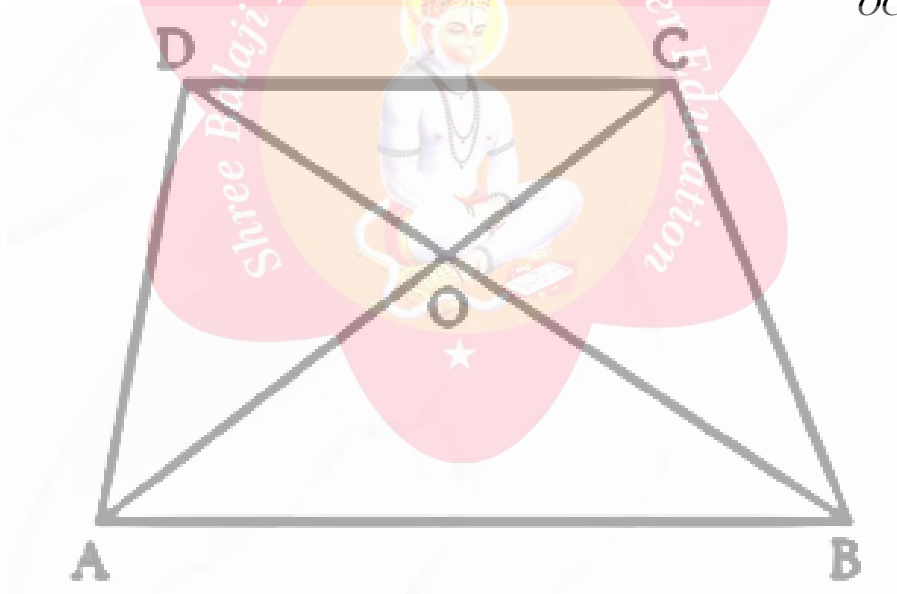
$$\angle ODC = \angle OBA$$

$$\Rightarrow \angle DCO = \angle OAB$$

$$\angle OAB = 55$$

**Q3.** Diagonals AC and BD of a trapezium ABCD with  $AB \parallel DC$  intersect each other at the point O. Using a similarity criterion for two triangles, show that  $\frac{OA}{OC} = \frac{OB}{OD}$ .

## Diagram



## Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

## Solution

In  $\triangle AOB$  and  $\triangle COD$

$$\angle AOB = \angle COD \quad (\text{vertically opposite angles})$$

$$\angle BAO = \angle DCO \quad (\text{alternate interior angles})$$

$$\Rightarrow \triangle AOB \sim \triangle COD \quad (\text{AA criterion})$$



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Hence,  $\frac{OA}{OC} = \frac{OB}{OD}$

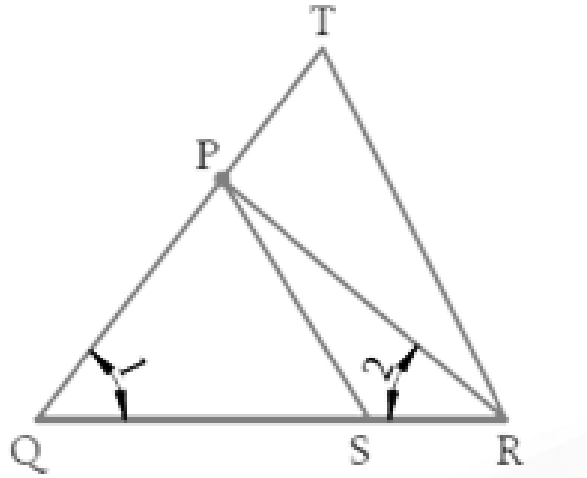




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**Q4.** In Figure 6.36  $\frac{QR}{QS} = \frac{QT}{PR}$  and  $\angle 1 = \angle 2$ . Show that  $\Delta PQS \sim \Delta TQR$ .

**Diagram**



**Reasoning:**

As we know if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

This is referred as SAS (Side – Angle – Side) similarity criterion for two triangles.

**Solution**

In  $\Delta PQR$

$$\angle 1 = \angle 2$$

$$\Rightarrow PR = PQ \quad (\text{In a triangle sides opposite to equal angles are equal})$$

In  $\Delta PQS$  and  $\Delta TQR$

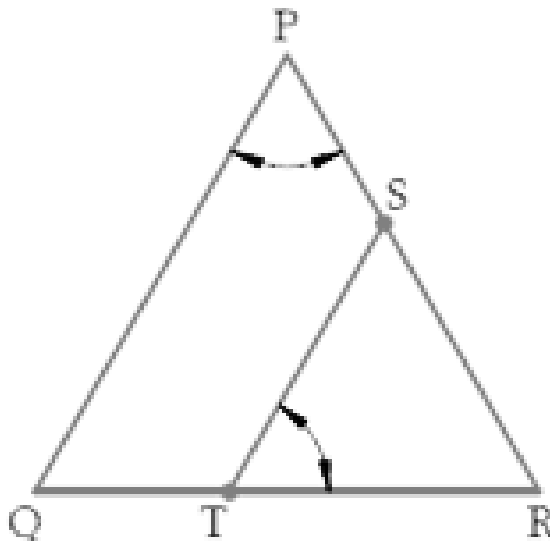
$$\angle PQS = \angle TQR = \angle 1 \quad (\text{same angle})$$

$$\frac{QR}{QS} = \frac{QT}{PQ} \quad (\because PR = PQ)$$

$$\Rightarrow \Delta PQS \sim \Delta TQR \quad (\text{SAS criterion})$$

**Q5.** S and T are points on sides PR and QR of  $\Delta PQR$  such that  $\angle P = \angle RTS$ . Show that  $\Delta RPQ \sim \Delta RTS$ .

**Diagram**





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## Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

## Solution

In  $\triangle RPQ$ ,  $\triangle RTS$

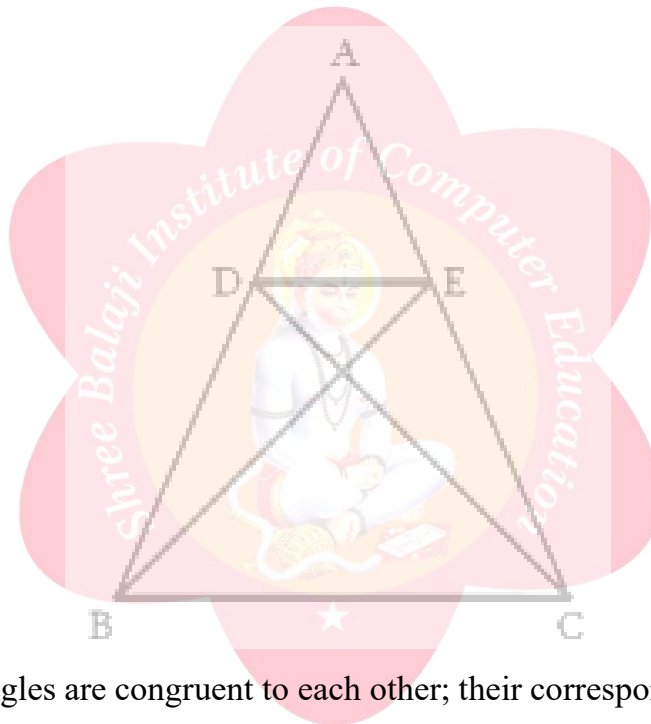
$$\angle RPQ = \angle RTS \quad (\text{given})$$

$$\angle PRQ = \angle TRS \quad (\text{common angle})$$

$$\Rightarrow \triangle RPQ \sim \triangle RTS \quad (\text{AA criterion})$$

**Q6.** In Figure 6.37, if  $\triangle ABE \cong \triangle ACD$ , show that  $\triangle ADE \sim \triangle ABC$ .

## Diagram



## Reasoning:

As we know if two triangles are congruent to each other; their corresponding parts are equal.

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

This is referred as SAS (Side – Angle – Side) similarity criterion for two triangles.

## Solution

In  $\triangle ABE$  and  $\triangle ACD$

$$AE = AD \quad (\triangle ABE \cong \triangle ACD \text{ given}) \dots\dots\dots (1)$$

$$AB = AC \quad (\triangle ABE \cong \triangle ACD \text{ given}) \dots\dots\dots (2)$$

Now Consider  $\triangle ADE$ ,  $\triangle ABC$

$$\frac{AD}{AB} = \frac{AE}{AC} \quad \text{from (1) \& (2)}$$

$$\text{and } \angle DAE = \angle BAC \quad (\text{Common angle})$$



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$\Rightarrow \Delta ADE \sim \Delta ABC$  (SAS criterion)



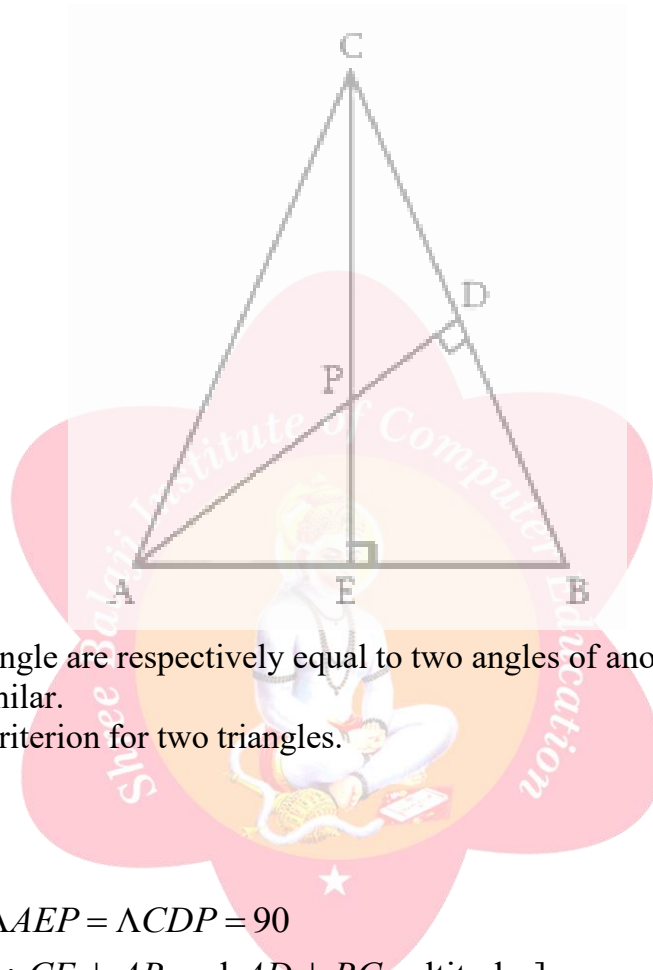


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**Q7.** In Figure 6.38, altitudes AD and CE of  $\triangle ABC$  intersect each other at the point P. Show that:

- (i)  $\triangle AEP \sim \triangle CDP$
- (ii)  $\triangle ABD \sim \triangle CBE$
- (iii)  $\triangle AEP \sim \triangle ADB$
- (iv)  $\triangle PDC \sim \triangle BEC$

## Diagram



### (i) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

### Solution:

In  $\triangle AEP$  and  $\triangle CDP$

$$\angle AEP = \angle CDP = 90^\circ$$

$$[\because CE \perp AB \text{ and } AD \perp BC; \text{ altitudes}]$$

$$\angle APE = \angle CPD \text{ (Vertically opposite angles)}$$

$$\Rightarrow \triangle AEP \sim \triangle CDP \text{ (AA criterion)}$$

### (ii) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

### Solution

In  $\triangle ABD$  and  $\triangle CBE$

$$\angle ADB = \angle CEB = 90^\circ$$

$$\angle ABD = \angle CBE \text{ (Common angle)}$$



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$\Rightarrow \triangle ABD \sim \triangle CBE$  (AA criterion)





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## (iii) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

## Solution

In  $\triangle AEP$  and  $\triangle ADB$

$$\angle AEP = \angle ADB = 90$$

$$\angle PAE = \angle BAD \text{ (Common angle)}$$

$$\Rightarrow \triangle AEP \sim \triangle ADB$$

## (iv) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar. This is referred as AA criterion for two triangles.

## Solution

In  $\triangle PDC$  and  $\triangle BEC$

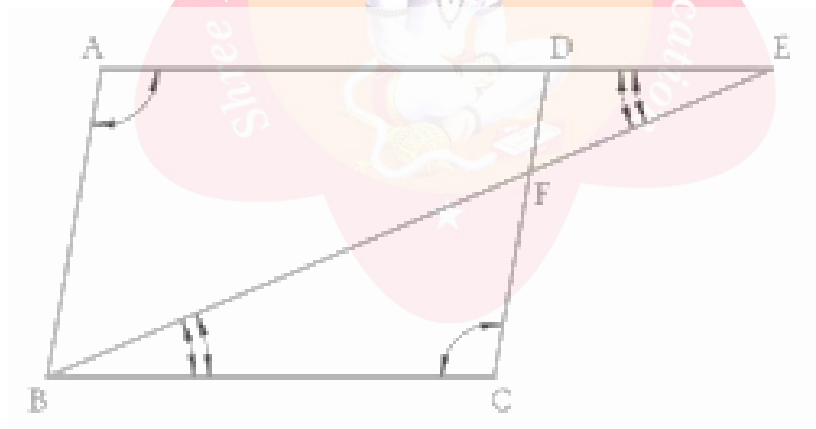
$$\angle PDC = \angle BEC = 90$$

$$\angle PCD = \angle BCE \text{ (Common angle)}$$

$$\Rightarrow \triangle PDC \sim \triangle BEC$$

**Q8.** E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that  $\triangle ABE \sim \triangle CFB$ .

## Diagram



## Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

## Solution

In  $\triangle ABE$  and  $\triangle CFB$

$$\angle BAE = \angle FCB \text{ (opposite angles of a parallelogram)}$$

$$\angle AEB = \angle FBC \text{ [ } AE \parallel BC \text{ and EB is a transversal, alternate interior angle]}$$

$$\Rightarrow \triangle ABE \sim \triangle CFE \text{ (AA criterion)}$$



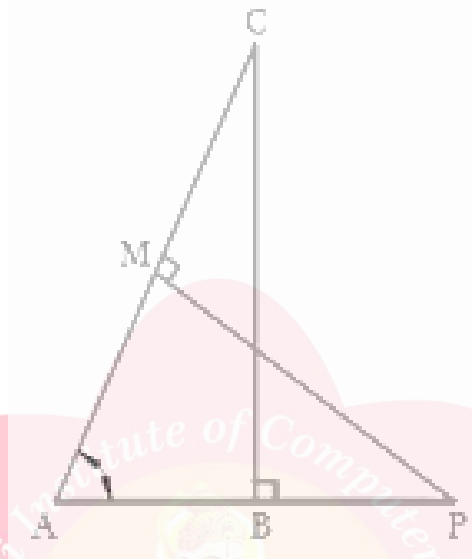
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**Q9.** In Figure 6.39,  $\triangle ABC$  and  $\triangle AMP$  are two right triangles, right angled at  $B$  and  $M$  respectively.

**Prove that:**

- (i)  $\triangle ABC \sim \triangle AMP$
- (ii)  $\frac{CA}{PA} = \frac{BC}{MP}$

**Diagram**



**(i) Reasoning:**

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar. This is referred as AA criterion for two triangles.

**Solution**

In  $\triangle ABC$  and  $\triangle AMP$

$$\begin{aligned}\angle ABC &= \angle AMP = 90^\circ \\ \angle BAC &= \angle MAP \text{ (Common angle)} \\ \Rightarrow \triangle ABC &\sim \triangle AMP\end{aligned}$$

**(ii) Reasoning:**

As we know that the ratio of any two corresponding sides in two equiangular triangles is always the same

**Solution**

In  $\triangle ABC$  and  $\triangle AMP$

$$\frac{CA}{PA} = \frac{BC}{MP} \quad [\because \triangle ABC \sim \triangle AMP]$$

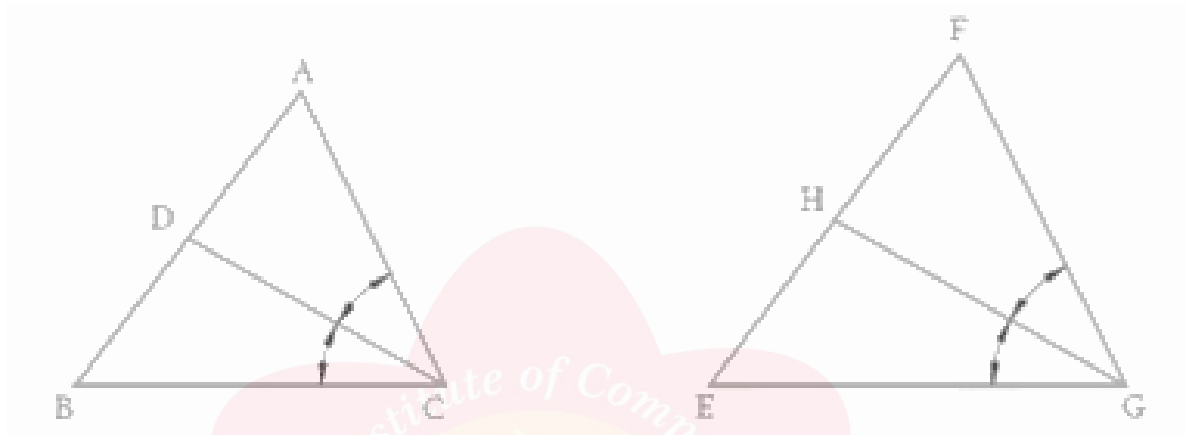


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**Q10.** CD and GH are respectively the bisectors of  $\angle ACB$  and  $\angle EGF$  such that D and H lie on sides AB and FE of  $\triangle ABC$  and  $\triangle EFG$  respectively. If  $\triangle ABC \sim \triangle FEG$ , show that:

- (i)  $\frac{CD}{GH} = \frac{AC}{FG}$
- (ii)  $\triangle DCB \sim \triangle HGE$
- (iii)  $\triangle DCA \sim \triangle HGF$

**Diagram**



**(i) Reasoning:**

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

**Solution**

$$\triangle ACB = \triangle FGE$$

$$\Rightarrow \frac{\angle ACB}{2} = \frac{\angle FGE}{2}$$

$$\Rightarrow \angle ACD = \angle FGH \quad (\text{CD and GH are bisectors of } \angle C \text{ and } \angle G \text{ respectively})$$

In  $\triangle ADC$  and  $\triangle FHG$

$$\angle DAC = \angle HFG \quad [ \triangle ADC \quad \triangle FEG ]$$

$$\angle ACD = \angle FGH$$

$$\Rightarrow \triangle ADC \sim \triangle FHG \quad (\text{AA criterion})$$

[If two triangles are similar, then their corresponding sides are in the same ratio]

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

**(ii) Reasoning:**

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.



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## Solution

In  $\triangle DCB$  and  $\triangle HGE$

$$\begin{aligned} \angle DBC &= \angle HEG \\ \angle DCB &= \angle HGE \\ \Rightarrow \triangle DCB &\sim \triangle HGE \quad \left[ \begin{array}{cc} \angle ABC & \angle FEG \\ \angle ACB & \angle FGE \end{array} \right] \\ &\quad \left[ \begin{array}{cc} 2 & 2 \end{array} \right] \\ &\quad \text{(AA criterion)} \end{aligned}$$

### (iii) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

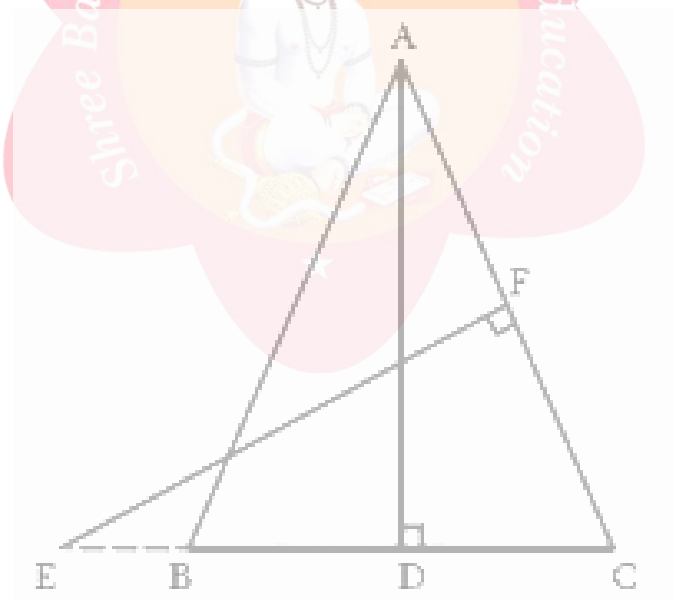
## Solution

In  $\triangle DAC$ ,  $\triangle HGF$

$$\begin{aligned} \angle DAC &= \angle HFG \\ \angle ACD &= \angle FGH \\ \Rightarrow \triangle DAC &\sim \triangle HGF \quad \left[ \begin{array}{cc} \angle ABC & \angle FEG \\ \angle ACB & \angle FGE \end{array} \right] \\ &\quad \left[ \begin{array}{cc} 2 & 2 \end{array} \right] \\ &\quad \text{(AA criterion)} \end{aligned}$$

**Q11.** In Figure 6.40, E is a point on side CB produced of an isosceles triangle ABC with  $AB = AC$ . If  $AD \perp BC$  and  $EF \perp AC$ , prove that  $\triangle ABD \sim \triangle ECF$ .

## Diagram



### Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

## Solution

In  $\triangle ABD$  and  $\triangle ECF$

$$\begin{aligned} \angle ADB &= \angle EFC = 90^\circ \\ \angle ABD &= \angle ECF \end{aligned} \quad \left[ \right]$$

$$\Rightarrow \triangle ABD \sim \triangle ECF$$



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$AD \perp BC$  and  $EF \perp AC$

[ In  $\triangle ABC$ ,  $AB = AC \Rightarrow \angle ABC = \angle ACB$ ]

(AA criterion)

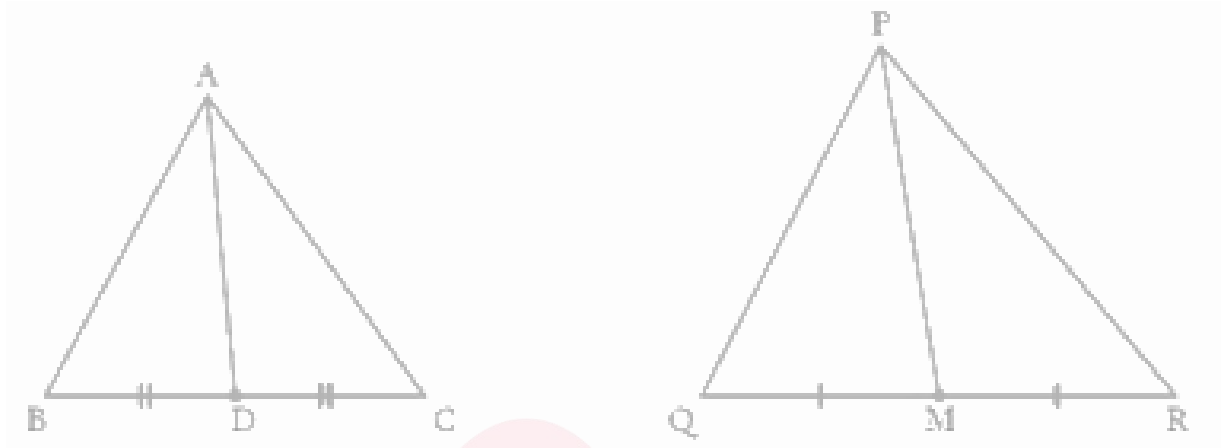




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**Q12.** Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ, QR and median PM of  $\Delta PQR$  (see Figure 6.41). Show that  $\Delta ABC \sim \Delta PQR$ .

## Diagram



## Reasoning:

As we know If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar. This is referred as SAS criterion for two triangles.

## Solution

In  $\Delta ABC$  and  $\Delta PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM} \quad [\text{given}]$$

AD and PM are median of  $\Delta ABC$  and  $\Delta PQR$  respectively

$$\Rightarrow \frac{BD}{QM} = \frac{BC}{QR} \cdot \frac{1}{2} = \frac{BC}{QR} \cdot \frac{1}{2}$$

Now In  $\Delta ABD$  and  $\Delta PQM$

$$\begin{aligned} \frac{AB}{PQ} &= \frac{BD}{QM} = \frac{AD}{PM} \\ \Rightarrow \Delta ABD &\sim \Delta PQM \end{aligned}$$

Now In  $\Delta ABC$  and  $\Delta PQR$

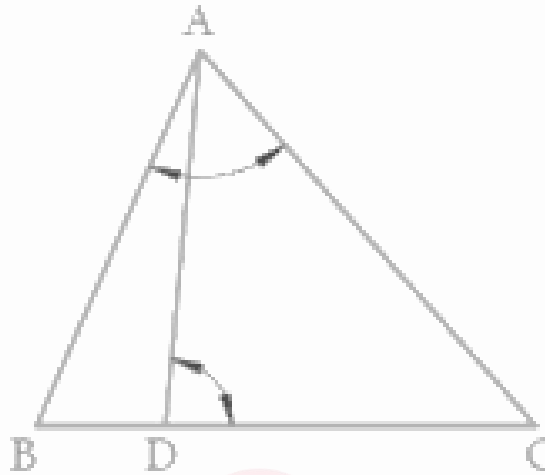
$$\begin{aligned} \frac{AB}{PQ} &= \frac{BC}{QR} && [\text{given in the statement}] \\ \angle ABC &= \angle PQR && [\because \Delta ABD \sim \Delta PQM] \\ \Rightarrow \Delta ABC &\sim \Delta PQR && [\text{SAS criterion}] \end{aligned}$$



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**Q13.** D is a point on the side BC of a triangle ABC such that  $\angle ADC = \angle BAC$ .  
Show that  $CA^2 = CB \cdot CD$ .

**Diagram**



**Reasoning:**

As we know that if two triangles are similar, then their corresponding sides are proportional.

**Solution**

In  $\triangle ABC$  and  $\triangle DAC$

$$\angle BAC = \angle ADC \quad (\text{Given in the statement})$$

$$\angle ACB = \angle ACD \quad (\text{Common angles})$$

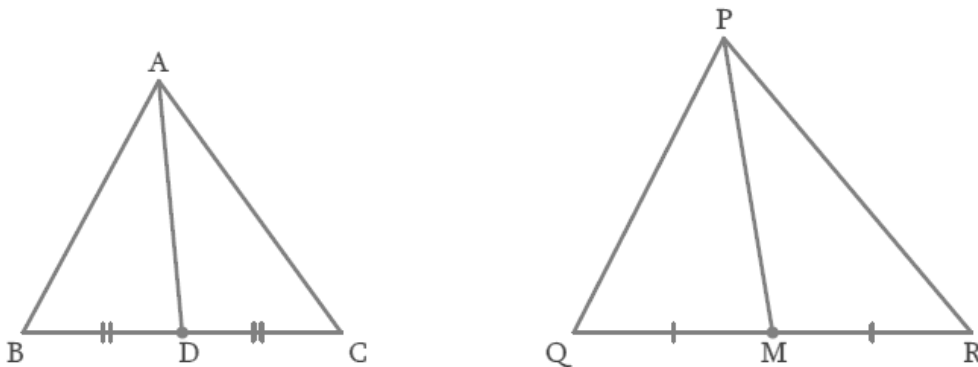
$$\Rightarrow \triangle ABC \sim \triangle DAC \quad (\text{AA criterion})$$

If two triangles are similar, then their corresponding sides are proportional

$$\Rightarrow \frac{CA}{CD} = \frac{CB}{CA}$$

$$\Rightarrow CA^2 = CB \cdot CD$$

**Q14.** Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR.  
Show that  $\triangle ABC \sim \triangle PQR$ .



**Reasoning:**

As we know If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.



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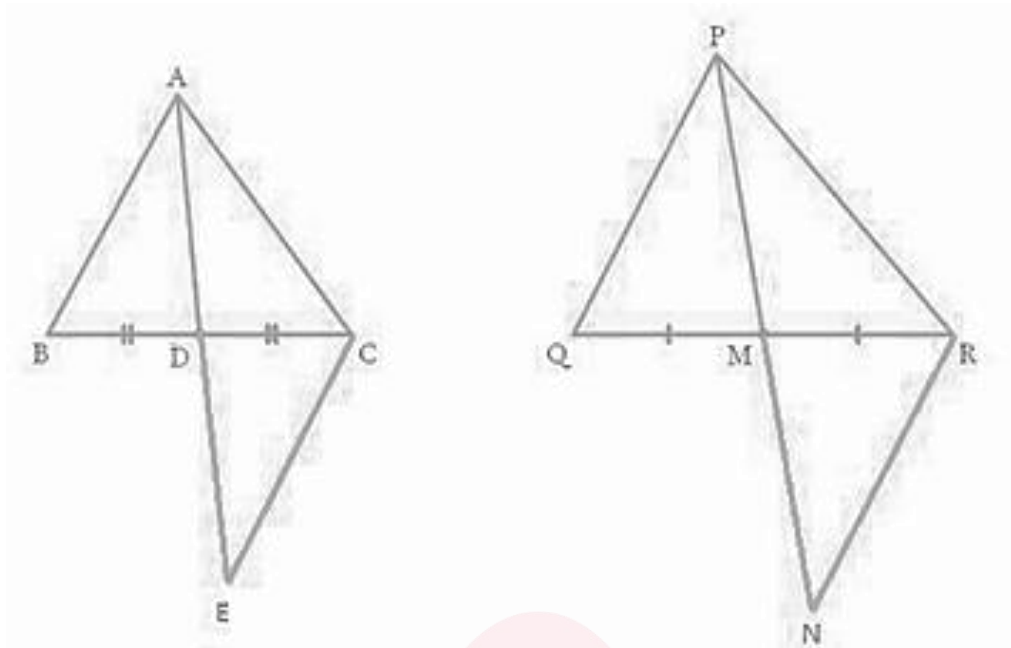
This is referred as SAS criterion for two triangles.





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Solution



Produce  $AD$  to  $E$  so that  $AD = DE$ . Join  $CE$

Similarly, produce  $PM$  to  $N$  such that  $PM = MN$ , and Join  $RN$ .

In  $\triangle ABD$  and  $\triangle CDE$

$$AD = DE$$

[By Construction]

$$BD = DC$$

[  $AP$  is the median]

$$\angle ADB = \angle CDE$$

[Vertically opposite angles]

$$\therefore \triangle ABD \cong \triangle CDE$$

[By SAS criterion of congruence]

$$\Rightarrow AB = CE$$

[CPCT]

...(i)

Also, in  $\triangle PQM$  and  $\triangle MNR$

$$PM = MN$$

[By Construction]

$$QM = MR$$

[  $PM$  is the median]

$$\angle PMQ = \angle MNR$$

[Vertically opposite angles]

$$\therefore \triangle PQM \cong \triangle MNR$$

[By SAS criterion of congruence]

$$\Rightarrow PQ = RN$$

[CPCT]

...(ii)

$$\text{Now, } \frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

[Given]

$$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AD}{PM}$$

[from (i) and (ii)]

$$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{2AD}{2PM}$$

$$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AE}{PN}$$

[  $2AD = AE$  and  $2PM = PN$ ]

$$\therefore \triangle ACE \sim \triangle PRN$$

[By SSS similarity criterion]



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Therefore,  $\angle CAE = \angle RPN$

Similarly,  $\angle BAE = \angle QPN$

$$\therefore \angle CAE + \angle BAE = \angle RPN + \angle QPN$$

$$\Rightarrow \angle BAC = \angle QPR$$

$$\Rightarrow \angle A = \angle P \quad \dots(\text{iii})$$

Now, In  $\triangle ABC$  and  $\triangle PQR$

$$\frac{AB}{PQ} = \frac{AC}{PR}$$

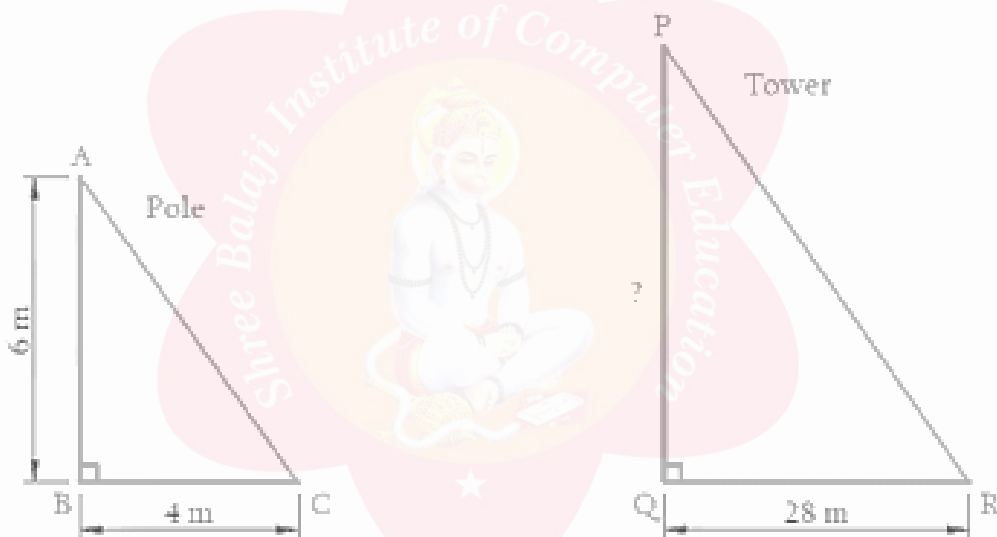
$$\frac{PQ}{PR}$$

$$\angle A = \angle P \quad [\text{from (iii)}]$$

$$\therefore \triangle ABC \sim \triangle PQR \quad [\text{By SAS similarity criterion}]$$

**Q15.** A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

**Diagram**



**Reasoning:**

The ratio of any two corresponding sides in two equiangular triangles is always the same.

**Solution**

AB is the pole = 6m

BC is the shadow of pole = 4m

PQ is the tower = ?

QR is the shadow of the tower = 28m

In  $\triangle ABC$  and  $\triangle PQR$

$$\angle ABC = \angle PQR = 90^\circ \quad (\text{The objects and shadows are perpendicular to each other})$$

$$\angle BAC = \angle QPR \quad (\text{Sunray fall on the pole and tower at the same angle, at the same time})$$



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$\Rightarrow \triangle ABC \quad \triangle PQR$  (AA criterion)





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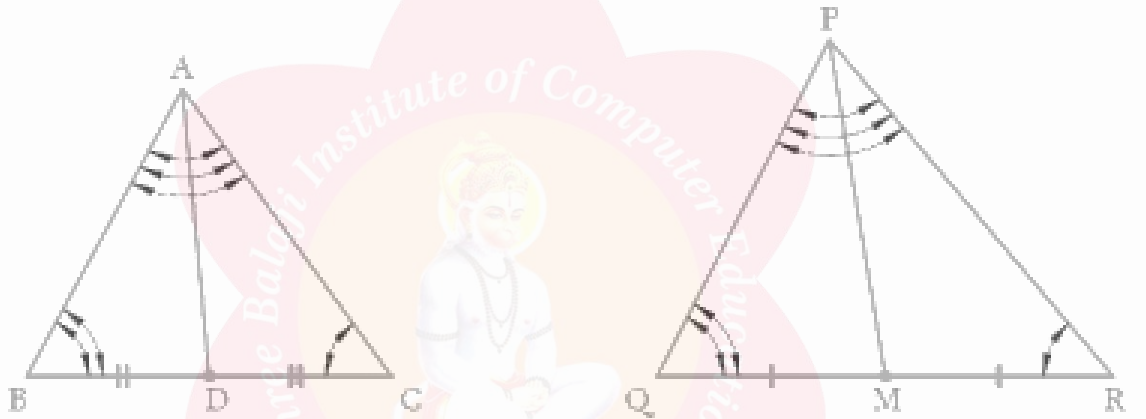
The ratio of any two corresponding sides in two equiangular triangles is always the same.

$$\begin{aligned}\Rightarrow \frac{AB}{BC} &= \frac{PQ}{QR} \\ \Rightarrow \frac{6m}{4m} &= \frac{PQ}{28m} \\ \Rightarrow PQ &= \frac{6 \times 28}{4} m \\ \Rightarrow PQ &= 42m\end{aligned}$$

Hence, the height of the tower is 42m.

**Q16.** If AD and PM are medians of triangles ABC and PQR, respectively where  $\triangle ABC \sim \triangle PQR$ , prove that  $\frac{AB}{PQ} = \frac{AD}{PM}$ .

**Diagram**



**Reasoning:**

As we know If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar. This is referred as SAS criterion for two triangles.

**Solution**

$$\begin{aligned}\triangle ABC &\sim \triangle PQR \\ \Rightarrow \angle ABC &= \angle PQR \quad (\text{corresponding angles})\end{aligned} \tag{1}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} \quad (\text{corresponding sides})$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} \times \frac{2}{2}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \quad (\text{D and M are mid-points of BC and QR}) \tag{2}$$



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In  $MED$  and  $MQM$

$$LABD=LPQM \quad (\text{from 1})$$

$$\frac{\overline{AB}}{\overline{PQ}} = \frac{\overline{BD}}{\overline{QM}} \quad (\text{from 2})$$

$$MBD-MQM \quad (\text{SAS criterion})$$

$$\frac{\overline{AB}}{\overline{PQ}} = \frac{\overline{BD}}{\overline{QM}} = \frac{\overline{AD}}{\overline{PM}} \quad (\text{corresponding sides})$$

$$\frac{\overline{AB}}{\overline{PQ}} = \frac{\overline{AD}}{\overline{PM}}$$



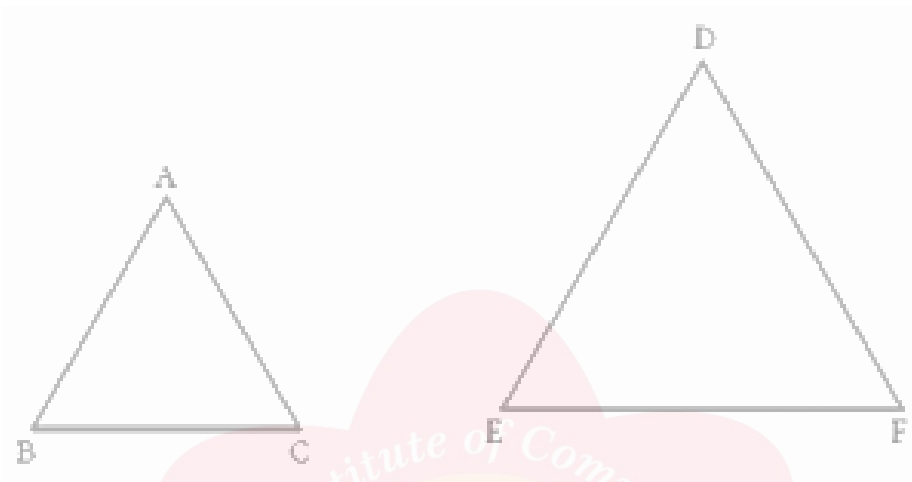


## Chapter - 6: Triangles

### Exercise 6.4 (Page 143)

**Q1.** Let  $\triangle ABC \sim \triangle DEF$  and their areas be, respectively,  $64 \text{ cm}^2$  and  $121 \text{ cm}^2$ . If  $EF = 15.4 \text{ cm}$ , find  $BC$ .

**Diagram**



**Reasoning:**

As we know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

**Solution**

$$\triangle ABC \sim \triangle DEF$$

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \left(\frac{BC}{EF}\right)^2$$

$$\frac{64 \text{ cm}^2}{121 \text{ cm}^2} = \left(\frac{BC}{15.4}\right)^2$$

$$64 \text{ cm}^2 = (BC)^2 \times \frac{121}{(15.4)^2}$$

$$(BC)^2 = \frac{121}{15.4^2} \times 64$$

$$BC = \frac{15.4 \times 8}{11}$$

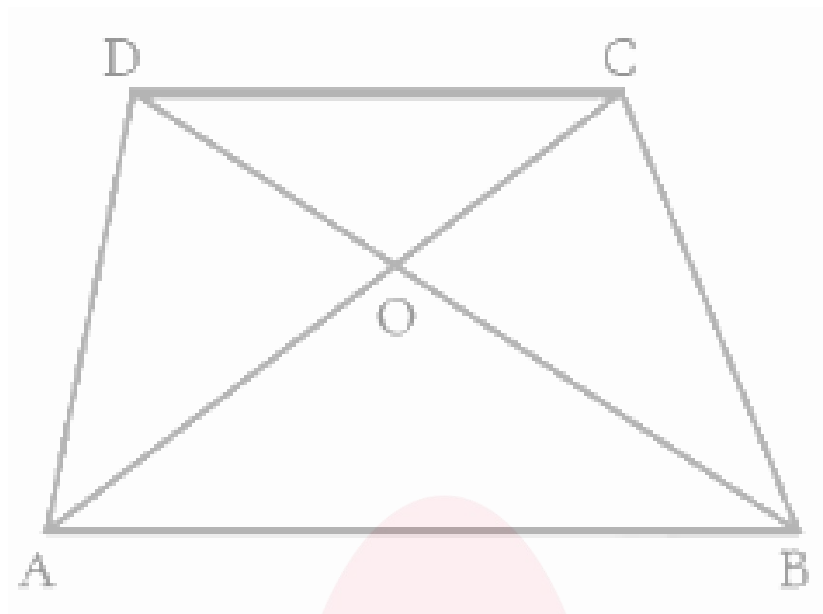
$$BC = 11.2 \text{ cm}$$



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**Q2.** Diagonals of a trapezium ABCD with  $AB \parallel DC$  intersect each other at the point O. If  $AB = 2 CD$ , find the ratio of the areas of triangles AOB and COD.

**Diagram**



**Reasoning:**

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

AA criterion.

**Solution**

In trapezium ABCD,  $AB \parallel CD$  and  $AB = 2CD$

Diagonals AC, BD intersect at 'O'

In  $\triangle AOB$  and  $\triangle COD$

$$\angle AOB = \angle COD \text{ (vertically opposite angles)}$$

$$\angle ABO = \angle CDO \text{ [alternate interior angles]}$$

$$\Rightarrow \triangle AOB \sim \triangle COD \text{ (AA criterion)}$$

$$\begin{aligned} \Rightarrow \frac{\text{Area of } \triangle AOB}{\text{Area of } \triangle COD} &= \left(\frac{AB}{CD}\right)^2 \quad [\text{theorem 6.6}] \\ &= \left(\frac{2CD}{CD}\right)^2 = \frac{4CD^2}{CD^2} = \frac{4}{1} \end{aligned}$$

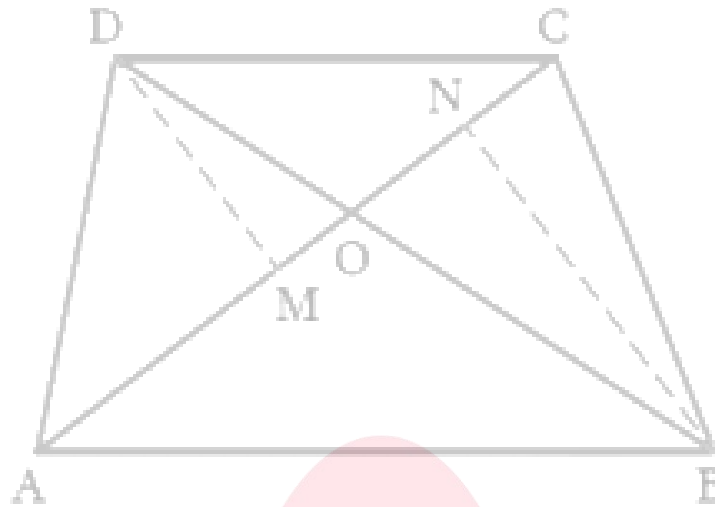
$$\Rightarrow \text{Area of } \triangle AOB : \text{area of } \triangle COD = 4:1$$



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**Q3.** In Fig. 6.44, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that  $\frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DBC)} = \frac{AO}{DO}$

**Diagram**



**Reasoning:**

AA criterion

**Solution:**

In  $\triangle ABC$

Draw  $AM \perp BC$

In  $\triangle DBC$

Draw  $DN \perp BC$

Now in  $\triangle AOM, \triangle DON$

$$\angle AMO = \angle DNO = 90^\circ$$

$$\angle AOM = \angle DON \text{ (Vertically opposite angles)}$$

$$\Rightarrow \triangle AOM \sim \triangle DON \text{ (AA criterion)}$$

$$\Rightarrow \frac{AM}{DN} = \frac{OM}{ON} = \frac{AO}{DO} \dots\dots\dots (1)$$

Now,

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times BC \times AM \\ \text{Area of } \triangle DBC &= \frac{1}{2} \times BC \times DN \end{aligned}$$



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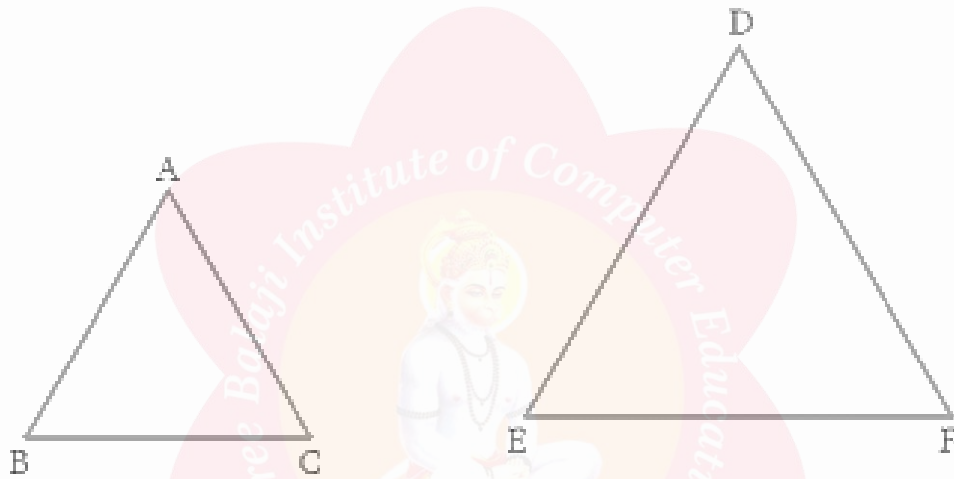
$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DBC} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times BC \times DN}$$

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DBC} = \frac{AM}{DN}$$

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DBC} = \frac{AO}{DO} \quad (\text{from (1)})$$

**Q4.** If the areas of two similar triangles are equal, prove that they are congruent.

**Diagram**



**Reasoning:**

As we know that two triangles are similar if their corresponding angles are equal and their corresponding sides are in the same ratio. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

As we know if three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

**Solution:**

$$\triangle ABC \sim \triangle DEF$$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} \quad (\text{SSS criterion})$$

But area of  $\triangle ABC = \text{area of } \triangle DEF$

$$\Rightarrow \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = 1 \dots\dots\dots(1)$$

But  $\text{Area of } \triangle ABC = (AB)^2 = (BC)^2 = (CA)^2$



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*Area of  $\triangle DEF$   $(DE)^2$   $(EF)^2$   $(FD)^2$*





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From (1)

$$\frac{(AB)^2}{(DE)^2} = \frac{(BC)^2}{(EF)^2} = \frac{(CA)^2}{(FD)^2} = 1$$

$$\Rightarrow \frac{(AB)^2}{(DE)^2} = 1$$

$$\Rightarrow (AB)^2 = (DE)^2$$

$$\Rightarrow AB = DE \dots \dots \dots (2)$$

Similarly,

$$\Rightarrow BC = EF \dots \dots \dots (3)$$

$$\Rightarrow CA = FD \dots \dots \dots (4)$$

Now, In  $\triangle ABC$  and  $\triangle DEF$

$$\Rightarrow AB = DE \quad \text{(from 2)}$$

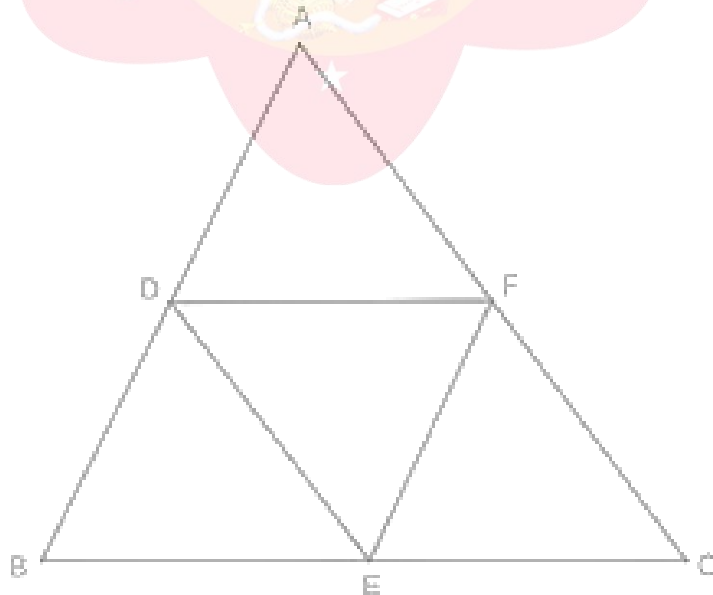
$$\Rightarrow BC = EF \quad \text{(from 3)}$$

$$\Rightarrow CA = FD \quad \text{(from 4)}$$

$$\Rightarrow \triangle ABC \cong \triangle DEF \text{ (SSS congruency)}$$

**Q5.** D, E and F are respectively the mid-points of sides AB, BC and CA of  $\triangle ABC$ . Find the ratio of the areas of  $\triangle DEF$  and  $\triangle ABC$ .

**Diagram**



**Reasoning:**

As we know that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and half of it – (mid-point theorem).



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## Solution

In  $\triangle ABC$ ,  $D$  and  $F$  are mid-points of  $AB$  and  $AC$  respectively.

$$\Rightarrow DF \parallel BC \text{ and } DF = \frac{1}{2} BC \quad (\text{by mid-point theorem})$$

Again,  $E$  is the mid-point of  $BC$

$$\Rightarrow DF \parallel BE \text{ and } DF = BE$$

In quadrilateral  $DFEB$

$$DF \parallel BE \text{ and } DF = BE$$

$\therefore DFEB$  is a parallelogram

$$\Rightarrow \angle B = \angle F \quad (1) \quad (\text{opposite angles of a parallelogram are equal})$$

Similarly, we can prove that,

$DFCE$  is a parallelogram

$$\Rightarrow \angle C = \angle D \quad (2) \quad (\text{opposite angles of a parallelogram are equal})$$

Now, In  $\triangle DEF$  and  $\triangle ABC$

$$\angle DFE = \angle ABC \quad (\text{from 1})$$

$$\angle EDF = \angle ACB \quad (\text{from 2})$$

$$\Rightarrow \triangle DEF \sim \triangle CAB \quad (\text{AA criterion})$$

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\frac{\text{Area of } \triangle DEF}{\text{Area of } \triangle ABC} = \frac{(DE)^2}{(AC)^2} = \frac{(EF)^2}{(AB)^2} = \frac{(DF)^2}{(BC)^2}$$

$$\begin{aligned} \frac{\text{Area of } \triangle DEF}{\text{Area of } \triangle ABC} &= \frac{(DF)^2}{(BC)^2} \\ &= \frac{\left(\frac{1}{2} BC\right)^2}{(BC)^2} \\ &= \frac{BC^2}{4BC^2} \\ &= \frac{1}{4} \end{aligned}$$



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The ratio of the areas of  $\triangle DEF$  and  $\triangle ABC$  is 1:4





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## Alternate method:

### Reasoning:

Theorem 8.9 midpoint theorem Pg. No.148.

### Solution:

In  $\triangle ABC$  D and E are midpoints of sides AB and AC

$$\Rightarrow DE \parallel BC \text{ and } DE = \frac{1}{2} BC \dots \dots \dots (1)$$

Now in quadrilateral DBFE

$$\Rightarrow DE \parallel BC \text{ and } DE = BF \text{ (from 1)}$$

$\Rightarrow$  DBFE is a parallelogram

$$\Rightarrow \text{Area of } \triangle DBF = \text{area of } \triangle DEF \dots \dots \dots (2)$$

( diagonal DF divides the parallelogram into two triangle of equal area)

Similarly, we can prove

$$\text{Area of } \triangle DBF = \text{Area of } \triangle EFC \dots \dots \dots (3)$$

$$\text{And area of } \triangle DEF = \text{Area of } \triangle ADE \dots \dots \dots (4)$$

From (2) ( 3) and (4)

$$\text{Area of } \triangle DBF = \text{Area of } \triangle DEF = \text{Area of } \triangle EFC = \text{Area of } \triangle ADE \dots \dots \dots (5)$$

(Things which are equal to the same thing are equal to one another – Euclid's 1<sup>st</sup> axiom.)

$$\text{Area of } \triangle ABC = \text{Area of } \triangle ADE + \text{Area of DBF} + \text{Area of } \triangle EFC + \text{Area of } \triangle DEF$$

From (5)

$$\text{Area of } \triangle ABC = 4 \times \text{Area of } \triangle DEF$$

$$\frac{\text{Area of } \triangle DEF}{\text{Area of } \triangle ABC} = \frac{1}{4}$$

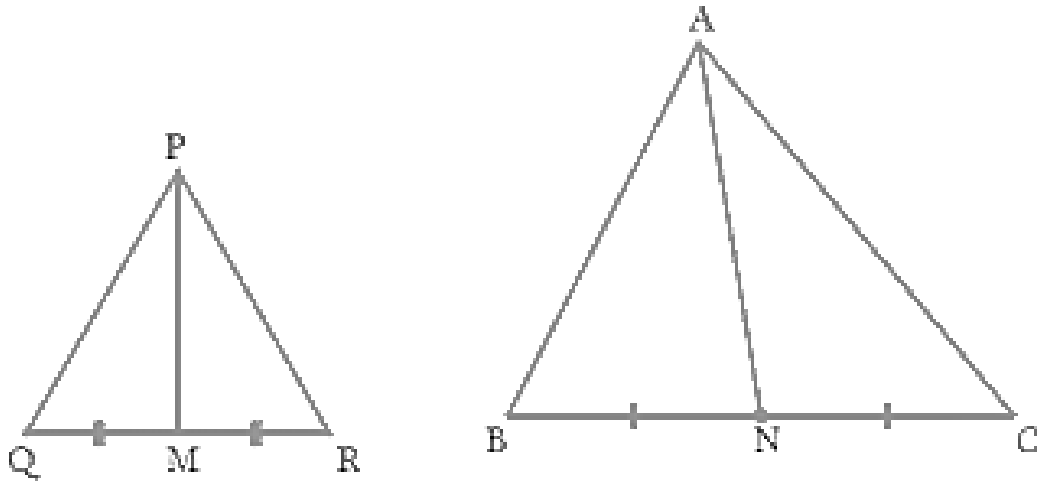
$$\text{Area of } \triangle DEF : \text{Area of } \triangle ABC = 1:4$$



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**Q6.** Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

**Diagram**



**Reasoning:**

As we know, if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar. And we know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

**Solution:**

In  $\Delta PQR$ , PM is the median and, In  $\Delta ABC$  AN is the median

$\Delta PQR \sim \Delta ABC$  (given)

$$\angle PQR = \angle ABC \dots\dots\dots (1)$$

$$\angle QPR = \angle BAC \dots\dots\dots (2)$$

$$\angle QRP = \angle BCA \dots\dots\dots (3)$$

$$\text{and } \frac{PQ}{AB} = \frac{QR}{BC} = \frac{RP}{CA} \dots\dots\dots (4)$$

( If two triangles are similar, then their corresponding angles are equal and corresponding sides are in the same ratio)

$$\frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta ABC} = \frac{(PQ)^2}{(AB)^2} = \frac{(QR)^2}{(BC)^2} = \frac{(RP)^2}{(CA)^2} \quad [\text{THEOREM 6.6}] \dots\dots\dots (5)$$

Now In  $\Delta PQM$  and  $\Delta ABN$

$$\angle PQM = \angle ABN \text{ (from 1)}$$

$$\text{And } \frac{PQ}{AB} = \frac{QM}{BN}$$



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$$\left[ \begin{array}{l} PQ = QR = 2QM; M, N \text{ mid points of } QR \text{ and } BC \\ \left| \begin{array}{ccc} AB & BC & 2BN \end{array} \right| \end{array} \right]$$

$\Rightarrow \Delta PQM \sim \Delta ABN$  [SAS similarly]

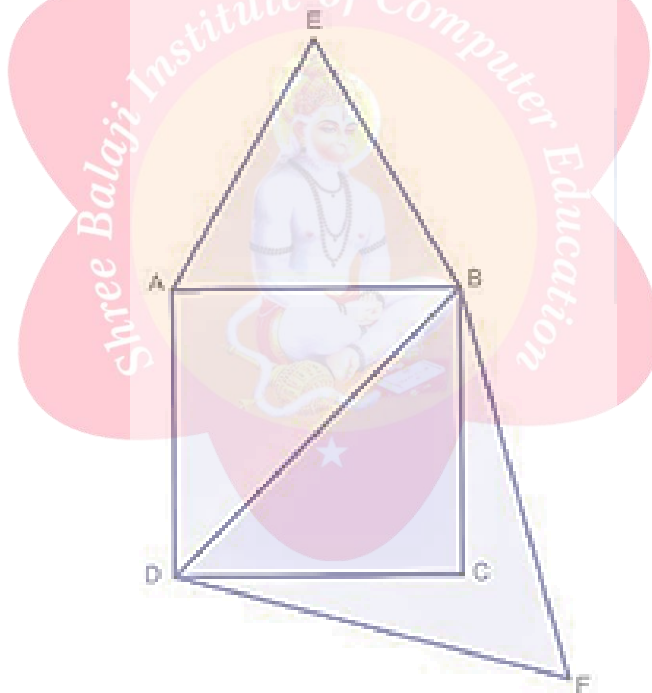
$$\Rightarrow \frac{\text{Area of } \Delta PQM}{\text{Area of } \Delta ABN} = \frac{(PQ)^2}{(AB)^2} = \frac{(QM)^2}{(BN)^2} = \frac{(PM)^2}{(AN)^2} \quad [ \text{theorem 6.6} ] \dots\dots (6)$$

From (5) and (6)

$$\frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta ABC} = \frac{(PM)^2}{(AN)^2}$$

**Q7.** Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

**Diagram**



**Reasoning:**

As we know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

**Solution:**

$\Delta ABE$  is described on the side AB of the square ABCD

$\Delta DBF$  is described on the diagonal BD of the square ABCD

**Since  $\Delta ABE$  and  $\Delta DBF$  are equilateral triangles**

$$\Delta ABE \sim \Delta DBF \quad [ \text{each angle in equilateral triangles is } 60^\circ ]$$



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The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\text{Area of } \triangle ABE = (AB)^2$$

$$\text{Area of } \triangle DBF = (DB)^2$$

$$\text{Area of } \triangle ABE = (AB)^2$$

$$\text{Area of } \triangle DBF = \left( \frac{2AB}{2} \right)^2 \quad [\text{diagonal of a square is } 2 \times \text{side}]$$

$$\text{Area of } \triangle ABE = AB^2$$

$$\text{Area of } \triangle DBF = 2AB^2$$

$$\text{Area of } \triangle ABE = 1$$

$$\text{Area of } \triangle DBF = 2$$

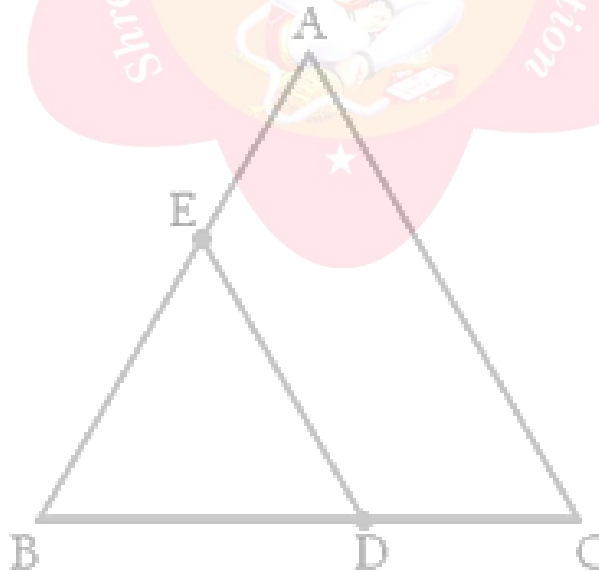
$$\text{Area of } \triangle ABE = \frac{1}{2} \times \text{Area of } \triangle DBF$$

Tick the correct answer and justify:

**Q8.** ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is

(A) 2 : 1 (B) 1 : 2 (C) 4 : 1 (D) 1 : 4

**Diagram**



**Reasoning:**

AAA criterion.

**Solution:**

$\triangle ABC \sim \triangle BDE$  ( equilateral triangles)



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The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides

$$\begin{aligned}\frac{\text{Area } \triangle ABC}{\text{Area } \triangle BDE} &= \frac{(BC)^2}{(BD)^2} \\ &= \frac{(BC)^2}{\left(\frac{BC}{2}\right)^2} \quad (\text{D is the midpoint of BC}) \\ &= \frac{(BC)^2 \times 4}{(BC)^2} \\ &= 4\end{aligned}$$

$$\text{Area } \triangle ABC : \text{Area } \triangle BDE = 4 : 1$$

**Answer (c)**

4:1

**Q9.** Sides of two similar triangles are in the ratio 4:9. Areas of these triangles are in the ratio

(A) 2:3      (B) 4:9      (C) 81:16      (D) 16:81

**Reasoning:**

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides

**Solution**

We know that,

Ratio of the areas of two similar triangles = square of the ratio of their corresponding sides

$$\begin{aligned}&= (4:9)^2 \\ &= 16:81\end{aligned}$$

**Answer (d)**

16:81



## Chapter - 6: Triangles

### Exercise 6.5(Page 150)

**Q1.** Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

- (i) 7 cm, 24 cm, 25 cm
- (ii) 3 cm, 8 cm, 6 cm
- (iii) 50 cm, 80 cm, 100 cm
- (iv) 13 cm, 12 cm, 5 cm

#### Reasoning:

As we know, in a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

#### Solution

(i)  $(25)^2 = 625$

$$\begin{aligned}7^2 + (24)^2 &= 49 + 576 \\&= 625 \\ \therefore (25)^2 &= 7^2 + (24)^2\end{aligned}$$

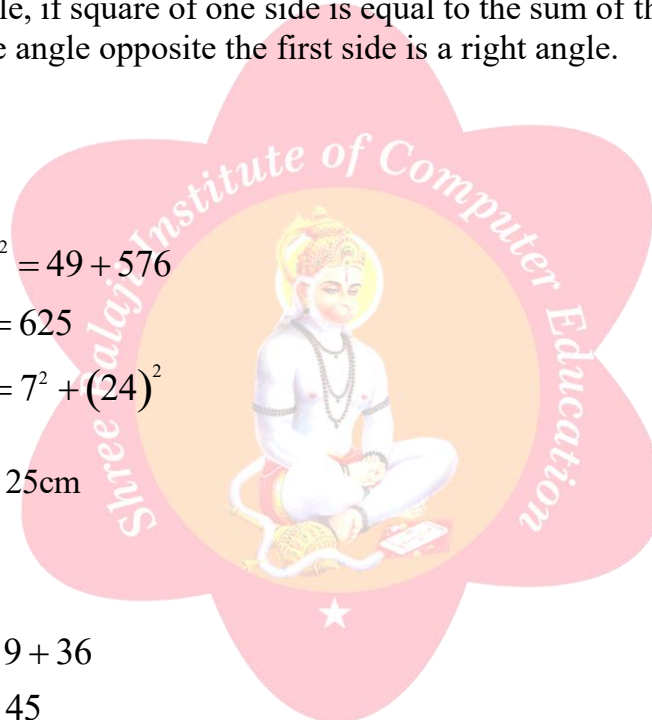
Length of hypotenuse = 25cm

(ii)  $8^2 = 64$

$$\begin{aligned}3^2 + 6^2 &= 9 + 36 \\&= 45 \\ 8^2 &\neq 3^2 + 6^2\end{aligned}$$

(iii)  $(100)^2 = 10000$

$$\begin{aligned}(50)^2 + (80)^2 &= 2500 + 6400 \\&= 8900 \\ (100)^2 &\neq (50)^2 + (80)^2\end{aligned}$$





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$$(iv) (13)^2 = 169$$

$$(12)^2 + 5^2 = 144 + 25 \\ = 169$$

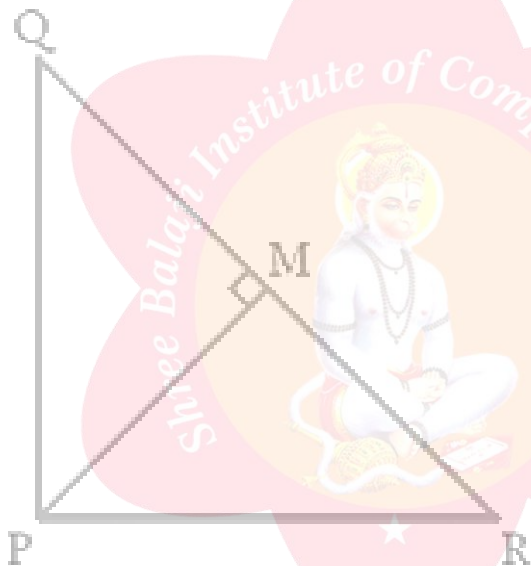
$$\therefore (13)^2 = (12)^2 + 5^2$$

Length of hypotenuse = 13cm

$\Rightarrow$  (i) and (iv) are right triangle.

**Q2.** PQR is a triangle right angled at P and M is a point on QR such that  $PM \perp QR$ . Show that  $(PM)^2 = QM \cdot MR$

**Diagram**



**Reasoning:**

As we know if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

**Solution**

In  $\triangle PQR$ ;  $\angle QPR = 90^\circ$  and  $PM \perp QR$

In  $\triangle PQR$  and  $\triangle MQP$

$$\angle QPR = \angle QMP = 90^\circ$$

$$\angle PRQ = \angle MQP \quad (\text{common angle})$$

$$\Rightarrow \triangle PQR \sim \triangle MQP \quad (\text{AA Similarity}) \quad (1)$$



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In  $\Delta PQR$  and  $\Delta MPR$

$$\angle QPR = \angle PMR = 90^\circ$$

$$\angle PRQ = \angle PRM \quad (\text{common angle})$$

$$\Rightarrow \Delta PQR \sim \Delta MPR \quad (\text{AA Similarity}) \quad (2)$$

From (1) and (2)

$$\Delta MQP \sim \Delta MPR$$

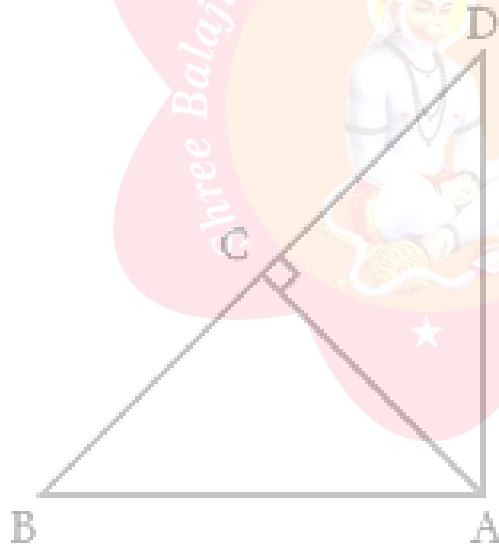
$$\frac{PM}{MR} = \frac{QM}{PM} \quad (\text{corresponding sides of similar triangles are proportional})$$

$$\Rightarrow PM^2 = QM.MR$$

**Q3.** In Fig. 6.53,  $ABD$  is a triangle right angled at  $A$  and  $AC \perp BD$ . Show that

- (i)  $AB^2 = BC.BD$
- (ii)  $AC^2 = BC.DC$
- (iii)  $AD^2 = BD.CD$

**Diagram**



**Reasoning:**

As we know if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

**Solution:**

- i). As we know if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

$$\Rightarrow \Delta BAD \sim \Delta BCA$$

$$\Rightarrow \frac{AB}{BD} = \frac{BC}{AB}$$



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(Corresponding sides of similar triangle)

$$\Rightarrow AB^2 = BC \cdot BD$$





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ii). As we know if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

$$\Rightarrow \triangle BCA \sim \triangle ACD$$

$$\Rightarrow \frac{AC}{CD} = \frac{BC}{AC} \quad (\text{Corresponding sides of similar triangle})$$

$$\Rightarrow AC^2 = BC \cdot DC$$

iii). As we know if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

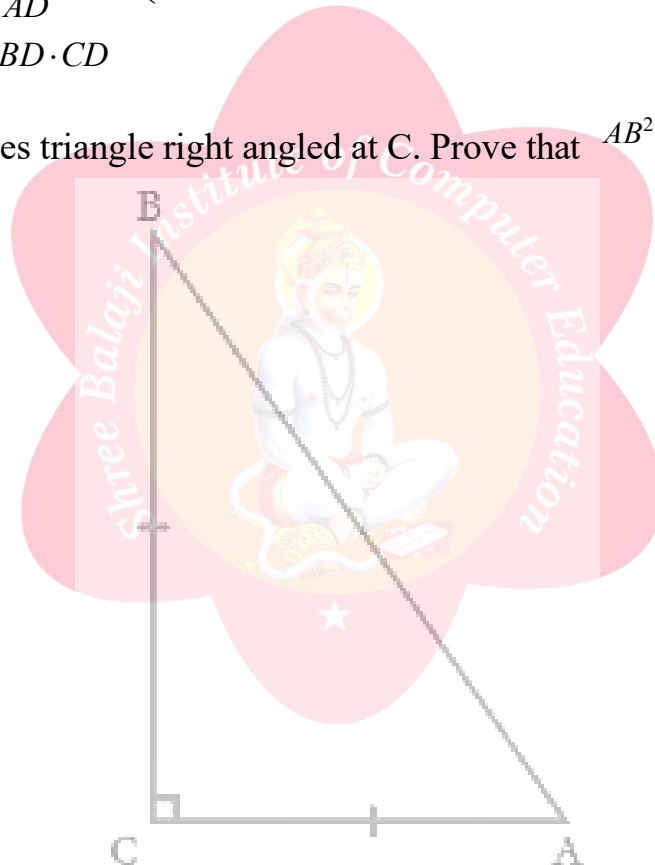
$$\Rightarrow \triangle BAD \sim \triangle ACD$$

$$\Rightarrow \frac{AD}{CD} = \frac{BD}{AD} \quad (\text{Corresponding sides of similar triangle})$$

$$AD^2 = BD \cdot CD$$

**Q4.** ABC is an isosceles triangle right angled at C. Prove that  $AB^2 = 2AC^2$ .

**Diagram**



**Reasoning:**

As we are aware, in a right triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides.

**Solution:**

In  $\triangle ABC$ ,  $\angle ACB = 90^\circ$  and  $AC = BC$

$$\text{But } AB^2 = AC^2 + BC^2$$

$$= AC^2 + AC^2 [\because AC = BC]$$

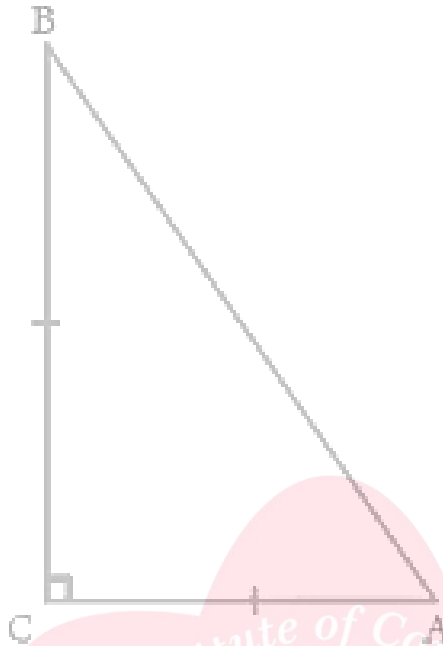
$$AB^2 = 2AC^2$$



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**Q5.** ABC is an isosceles triangle with  $AC = BC$ . If  $AB^2 = 2AC^2$ , prove that ABC is a right triangle.

**Diagram**



**Reasoning:**

As we know, in a triangle, if square of one side is equal to the sum of the square of the other two sides then the angle opposite the first side is a right angle.

**Solution**

In  $\triangle ABC$

$$AC = BC$$

$$\text{And } AB^2 = 2AC^2$$

$$= AC^2 + AC^2$$

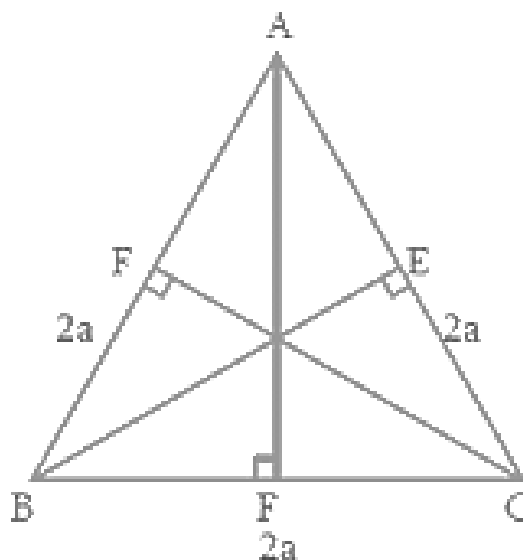
$$AB^2 = AC^2 + BC^2 \quad [AC = BC]$$

$$\Rightarrow \angle ACB = 90^\circ$$

$$\Rightarrow \triangle ABC \text{ is a right triangle}$$

**Q6.** ABC is an equilateral triangle of side  $2a$ . Find each of its altitudes.

**Diagram**





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## Reasoning:

We know that in an equilateral triangle perpendicular drawn from vertex to the opposite side, bisects the side.

As we know that, in a right triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides

## Solution

In  $\triangle ABC$

$$AB = BC = CA = 2a$$

$$AD \perp BC$$

$$\Rightarrow BD = CD = \frac{1}{2} BC = a$$

In  $\triangle ADB$

$$AB^2 = AD^2 + BD^2$$

$$AD^2 = AB^2 - BD^2$$

$$= (2a)^2 - a^2$$

$$= 4a^2 - a^2$$

$$= 3a^2$$

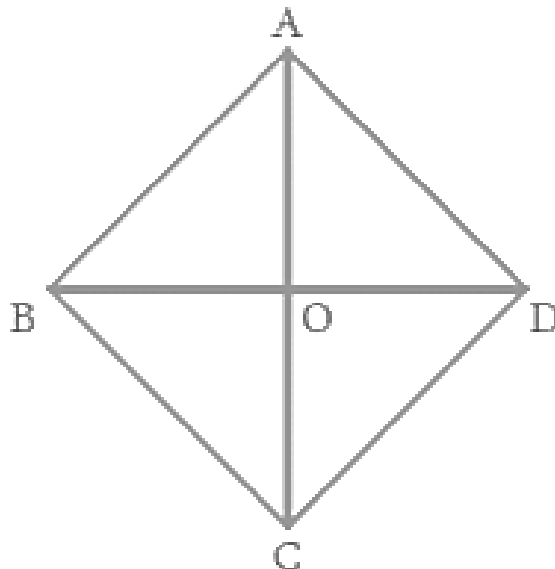
$$AD = \sqrt{3}a$$

$$\Rightarrow AD = \sqrt{3}a \text{ units}$$

Similarly, we can prove that,  $BE = CF = \sqrt{3}a$  units

**Q7.** Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

## Diagram





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## Reasoning:

As we know, in a rhombus, diagonals bisect each other perpendicularly.

## Solution:

In rhombus ABCD

$$AC \perp BD \text{ and } OA = OC; OB = OD$$

In  $\triangle AOB$

$$\angle AOB = 90^\circ$$

$$\Rightarrow AB^2 = OA^2 + OB^2 \dots\dots\dots (1)$$

Similarly, we can prove

$$BC^2 = OB^2 + OC^2 \dots\dots\dots (2)$$

$$CD^2 = OC^2 + OD^2 \dots\dots\dots (3)$$

$$AD^2 = OD^2 + OA^2 \dots\dots\dots (4)$$

Adding (1), (2), (3) and (4)

$$AB^2 + BC^2 + CD^2 + AD^2 = OA^2 + OB^2 + OB^2 + OC^2 + OC^2 + OD^2 + OD^2 + OA^2$$

$$AB^2 + BC^2 + CD^2 + AD^2 = 2OA^2 + 2OB^2 + 2OC^2 + 2OD^2$$

$$AB^2 + BC^2 + CD^2 + AD^2 = 2[OA^2 + OB^2 + OC^2 + OD^2]$$

$$AB^2 + BC^2 + CD^2 + AD^2 = 2 \left[ \left( \frac{AC}{2} \right)^2 + \left( \frac{BD}{2} \right)^2 + \left( \frac{AC}{2} \right)^2 + \left( \frac{BD}{2} \right)^2 \right]$$

$$\left[ \begin{array}{l} OA = OC = \frac{AC}{2} \text{ and } OB = OD = \frac{BD}{2} \end{array} \right]$$

$$AB^2 + BC^2 + CD^2 + AD^2 = 2 \left[ \frac{AC^2}{2} + \frac{BD^2}{2} + \frac{AC^2}{2} + \frac{BD^2}{2} \right]$$

$$AB^2 + BC^2 + CD^2 + AD^2 = 2 \left[ \frac{4}{2} \left[ \frac{AC^2}{2} + \frac{BD^2}{2} \right] \right]$$

$$AB^2 + BC^2 + CD^2 + AD^2 = 4 \left[ \frac{4}{4} \left[ \frac{AC^2}{2} + \frac{BD^2}{2} \right] \right]$$

$$AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$$



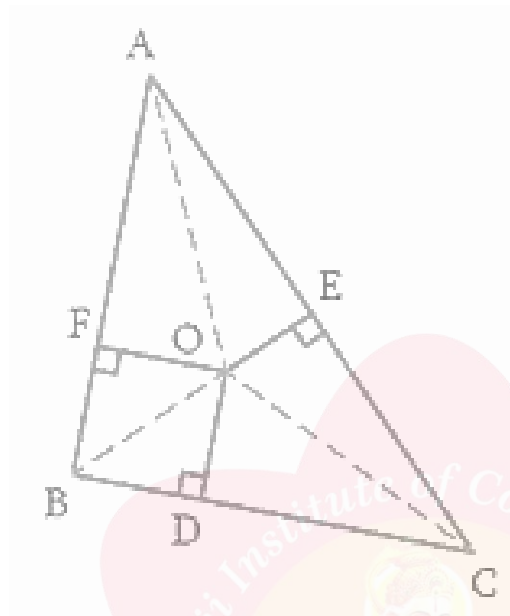
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**Q8.** In Figure 6.54, O is a point in the interior of a triangle ABC,  $OD \perp BC$ ,  $OE \perp AC$  and  $OF \perp AB$ . Show that

i.  $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

ii.  $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

**Diagram**



**Reasoning:**

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

**Solution:**

(i) In  $\triangle ABC$

$OD \perp BC, OE \perp AC$  and  $OF \perp AB$

OA, OB and OC joined

In  $\triangle OAF$

$$OA^2 = AF^2 + OF^2 \quad [\angle OFA = 90^\circ] \dots\dots\dots (1)$$

Similarly, In  $\triangle OBD$

$$OB^2 = BD^2 + OD^2 \quad [\angle ODB = 90^\circ] \dots\dots\dots (2)$$

In  $\triangle OCE$

$$OC^2 = CE^2 + OE^2 \quad [\angle OEC = 90^\circ] \dots\dots\dots (3)$$

Adding (1), (2) and (3)

$$OA^2 + OB^2 + OC^2 = AF^2 + OF^2 + BD^2 + OD^2 + CE^2 + OE^2$$



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$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2 \dots\dots\dots(4)$$





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(ii) From (4)

$$(OA^2 - OE^2) + (OB^2 - OF^2) + (OC^2 - OD^2) = AF^2 + BD^2 + CE^2$$

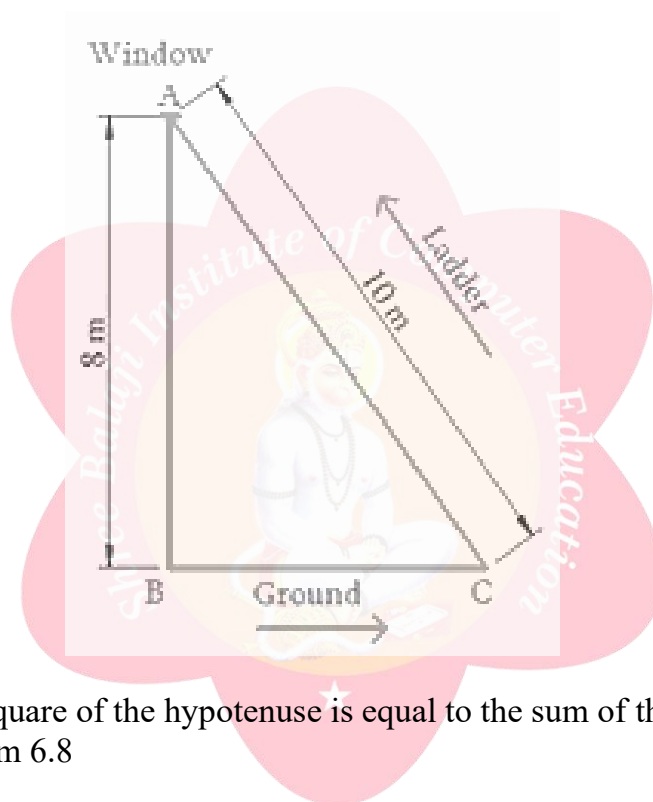
(Rearranging the left side terms)

$$AE^2 + BF^2 + CD^2 = AF^2 + BD^2 + CE^2$$

[  $\triangle OAE, \triangle OBD$  and  $\triangle OCE$  are right triangles]

**Q9.** A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

**Diagram**



**Reasoning:**

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

**Solution:**

AB is height of the windows from the ground = 8m

AC is the length of the ladder = 10m

BC is the foot of the ladder from the base of ground = ?

Since  $\triangle ABC$  is right angled triangle ( $\angle ABC = 90^\circ$ )

$$BC^2 = AC^2 - AB^2 \quad (\text{Pythagoras theorem})$$

$$BC^2 = 10^2 - 8^2$$

$$BC^2 = 100 - 64$$

$$BC^2 = 36$$



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$$BC = 6\text{ m}$$

The distance of the foot of the ladder from the base of the wall is 6m

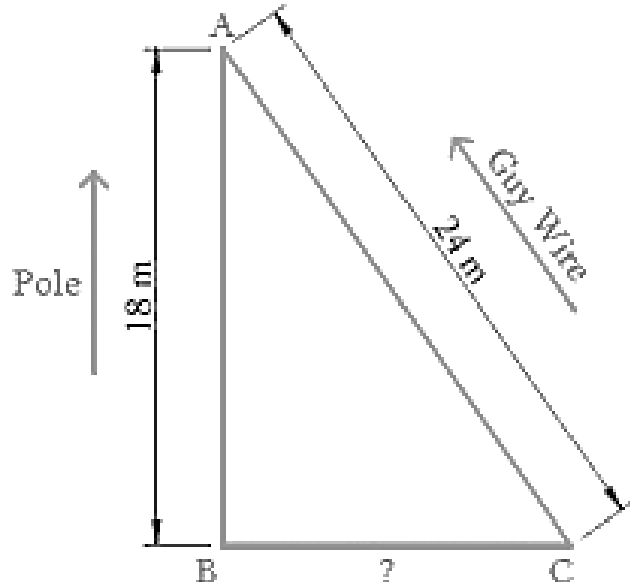




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**Q10.** A guy wire attached to a vertical pole of height 18m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

## Diagram



## Reasoning:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

## Solution

AB is the length of the pole = 18m

AC is the length of the guy wire = 24m

BC is the distance of the stake from the pole = ? ★

In  $\triangle ABC$   $\angle ABC = 90^\circ$

$$BC^2 = AC^2 - AB^2 \quad (\text{Pythagoras theorem})$$

$$BC^2 = 24^2 - 18^2$$

$$BC^2 = 576 - 324$$

$$BC^2 = 252$$

$$BC = 2 \times 3 \quad 7$$

$$BC = 6 \quad 7$$

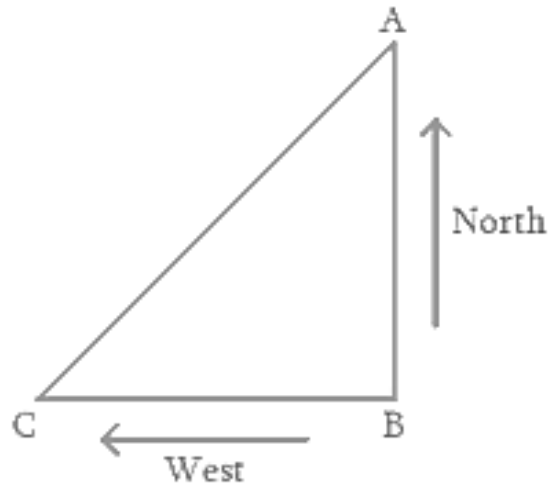
The distance of the stake from the pole is  $6 \quad 7m$



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**Q11.** An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after  $1\frac{1}{2}$  hours?

**Diagram**



**Reasoning:**

We have to find the distance travelled by aeroplanes, we need to use

$$\text{distance} = \text{speed} \times \text{time}$$

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

**Solution:**

AB is the distance travelled by aeroplane travelling towards north

$$\begin{aligned} AB &= 1000 \text{ km/hr} \times 1\frac{1}{2} \text{ hr} \\ &= 1000 \times \frac{3}{2} \text{ km} \\ AB &= 1500 \text{ km} \end{aligned}$$

BC is the distance travelled by another aeroplane travelling towards south

$$\begin{aligned} BC &= 1200 \text{ km/hr} \times 1\frac{1}{2} \text{ hr} \\ &= 1200 \times \frac{3}{2} \text{ hr} \\ BC &= 1800 \text{ km} \end{aligned}$$



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Now, In  $\triangle ABC$ ,  $\angle ABC = 90^\circ$

$$AC^2 = AB^2 + BC^2 \text{ (Pythagoras theorem)}$$

$$= (1500)^2 + (1800)^2$$

$$= 2250000 + 3240000$$

$$AC^2 = 5490000$$

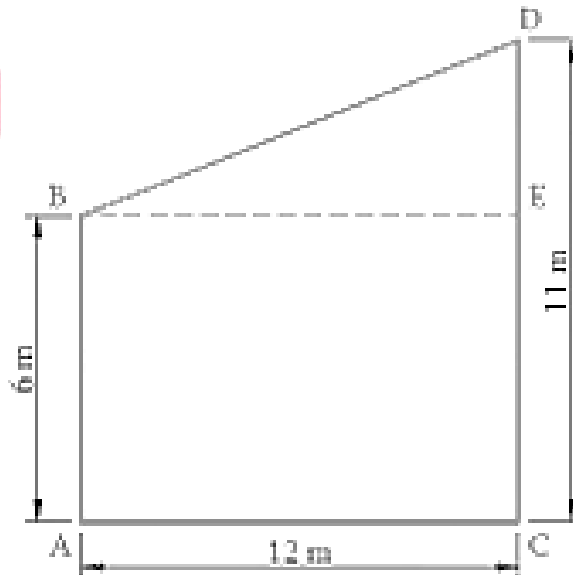
$$AC = \sqrt{5490000}$$

$$= 300\sqrt{61} \text{ km}$$

The distance between two planes after  $1\frac{1}{2}$  hr =  $300\sqrt{61} \text{ km}$

**Q12.** Two poles of heights 6 m and 11 m stand on plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

**Diagram**



**Reasoning:**

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

**Solution:**

AB is the height of one pole = 6m

CD is the height of another pole = 11m

AC is the distance between two poles at bottom = 12m

BD is the distance between the tops of the poles = ?

Draw  $BE \parallel AC$



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Now consider, In  $\triangle BED$

$$\angle BED = 90^\circ$$

$$BE = AC = 12 \text{ m}$$

$$DE = CD - CE$$

$$DE = 11 - 6 = 5 \text{ cm}$$

Now

$$BD^2 = BE^2 + DE^2 \quad (\text{Pythagoras theorem})$$

$$= 12^2 + 5^2$$

$$= 144 + 25$$

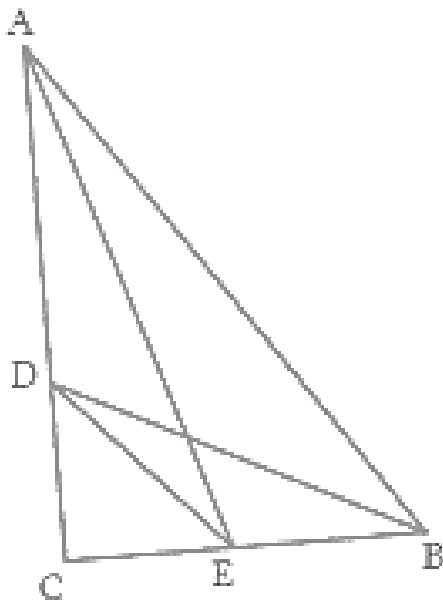
$$BD^2 = 169$$

$$BD = 13 \text{ m}$$

The distance between the tops of poles = 13m

**Q13.** D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that  $AE^2 + BD^2 = AB^2 + DE^2$ .

**Diagram**



**Reasoning:**

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

**Solution**

In  $\triangle ABC$ ,  $\angle ACB = 90^\circ$

D, E are points on AC, BC



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Join AE, DE and BD





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In  $\triangle ACE$

$$AE^2 = AC^2 + CE^2 \text{ (Pythagoras theorem) .....(1)}$$

In  $\triangle DCB$

$$BD^2 = CD^2 + BC^2 \text{ ..... (2)}$$

Adding (1) and (2)

$$\begin{aligned} AE^2 + BD^2 &= AC^2 + CE^2 + CD^2 + BC^2 \\ &= AC^2 + BC^2 + EC^2 + CD^2 \\ &= AB^2 + DE^2 \end{aligned}$$

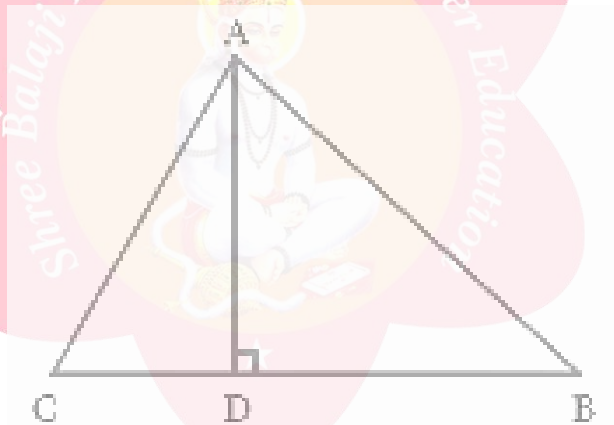
$$[In \triangle ABC, \angle C = 90^\circ \Rightarrow AC^2 + BC^2 = AB^2 \text{ and}]$$

$$In \triangle CDE, \angle DCE = 90^\circ \Rightarrow CD^2 + CE^2 = DE^2]$$

$$\Rightarrow AE^2 + BD^2 = AB^2 + DE^2$$

**Q14.** The perpendicular from A on side BC of a  $\triangle ABC$  intersects BC at D such that  $DB = 3CD$  (see Fig. 6.55). Prove that  $2AB^2 = 2AC^2 + BC^2$ .

**Diagram**



**Reasoning:**

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

**Solution:**

In  $\triangle ABC$ ,  $AD \perp BC$  and  $BD = 3CD$

$$BD + CD = BC$$

$$3CD + CD = BC$$

$$4CD = BC$$

$$CD = \frac{1}{4} BC \text{ .....(1)}$$

$$\text{and, } BD = \frac{3}{4} BC \text{ .....(2)}$$



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In  $\triangle ADC$

$$AC^2 = AD^2 + CD^2 \quad [\angle ADC = 90^\circ]$$

$$AD^2 = AC^2 - CD^2 \quad \dots\dots\dots(3)$$

In  $\triangle ADB$

$$AB^2 = AD^2 + BD^2 \quad [\angle ADB = 90^\circ]$$

$$AB^2 = AC^2 - CD^2 + BD^2 \quad [\text{from (3)}]$$

$$AB^2 = AC^2 + \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 \quad [\text{from (1) and (2)}]$$

$$AB^2 = AC^2 + \frac{9BC^2 - BC^2}{16}$$

$$AB^2 = AC^2 + \frac{8BC^2}{16}$$

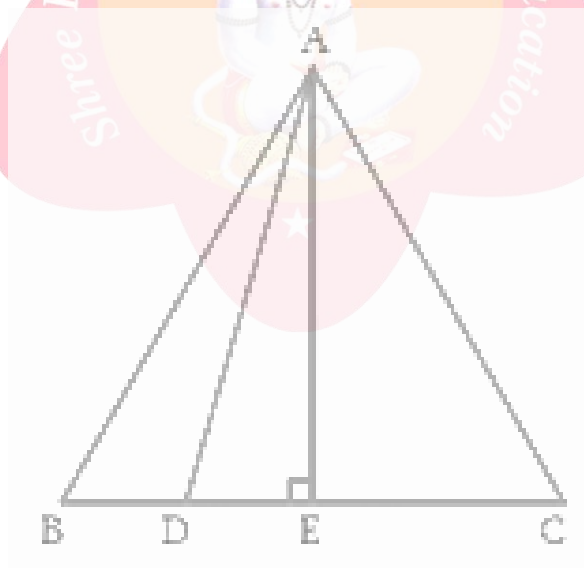
$$AB^2 = AC^2 + \frac{1}{2}BC^2$$

$$2AB^2 = 2AC^2 + BC^2$$

**Q15.** In an equilateral triangle ABC, D is a point on side BC such that  $BD = \frac{1}{3}BC$

Prove that  $9AD^2 = 7AB^2$ .

**Diagram**



**Reasoning:**

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

**Solution:**

In  $\triangle ABC$ ;  $AB = BC = CA$  and  $BD = \frac{1}{3}BC$

Draw  $AE \perp BC$



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$$BE = CE = \frac{1}{2} BC$$





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[ In an equilateral triangle perpendicular drawn from vertex to opposite side bisects the side]

Now In  $\triangle ADE$

$$AD^2 = AE^2 + DE^2 \text{ (Pythagoras theorem)}$$

$$= \left( \frac{3}{2} BC \right)^2 + (BE - BD)^2$$

[ AE is the height of an equilateral triangle which is equal to  $\frac{3}{2}$  side]

$$AD^2 = \frac{3}{4} BC^2 + \left[ \frac{BC}{2} - \frac{BC}{2} \right]^2$$

$$AD^2 = \frac{3}{4} BC^2 + \left( \frac{BC}{2} \right)^2$$

$$AD^2 = \frac{3}{4} BC^2 + \frac{BC^2}{4}$$

$$AD^2 = \frac{27BC^2 + BC^2}{36}$$

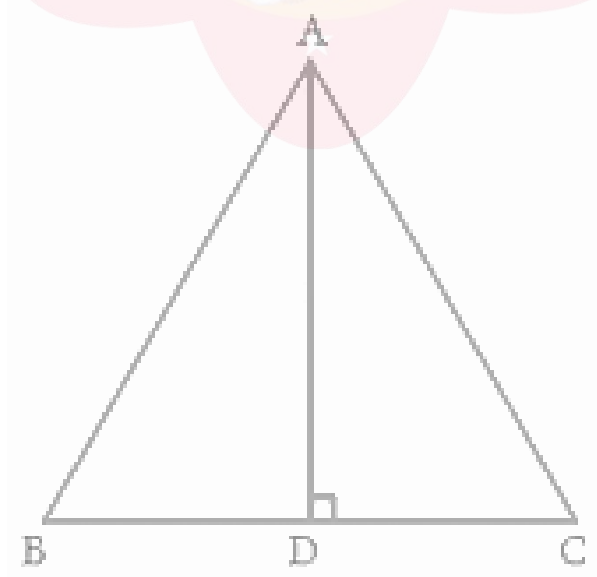
$$36AD^2 = 28BC^2$$

$$9AD^2 = 7BC^2$$

$$9AD^2 = 7AB^2 \text{ [ } AB = BC = CA \text{]}$$

**Q16.** In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

**Diagram**



We have to prove  $3BC^2 = 4AD^2$

**Reasoning:**



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In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8





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## Solution

In  $\triangle ABC$

$$AB = BC = CA$$

$$AD \perp BC \Rightarrow BD = CD = \frac{BC}{2}$$

Now In  $\triangle ADC$

$$AC^2 = AD^2 + CD^2$$

$$BC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 \left[ AC = BC \text{ and } CD = \frac{BC}{2} \right]$$

$$BC^2 = AD^2 + \frac{BC^2}{4}$$

$$BC^2 - \frac{BC^2}{4} = AD^2$$

$$\frac{3BC^2}{4} = AD^2$$

$$3BC^2 - 4AD^2$$

**Q17.** Tick the correct answer and justify: In  $\triangle ABC$ ,  $AB = 6\sqrt{3}$  cm,  $AC = 12$  cm and  $BC = 6$  cm. The angle B is

(A)  $120^\circ$

(B)  $60^\circ$

(C)  $90^\circ$

(D)  $45^\circ$

### Reasoning:

In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle. Theorem 6.9

### Solution:

(c)

In  $\triangle ABC$

$$AB = 6\sqrt{3} \text{ cm}; AC = 12 \text{ cm}; BC = 6 \text{ cm}$$

$$AB^2 = 108 \text{ cm}^2; AC^2 = 144 \text{ cm}^2; BC^2 = 36 \text{ cm}^2$$

$$AB^2 + BC^2 = (108 + 36) \text{ cm}^2$$

$$= 144 \text{ cm}^2$$

$$\Rightarrow AC^2 = AB^2 + BC^2$$

Pythagoras theorem satisfied



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$$\Rightarrow \angle ABC = 90^\circ$$

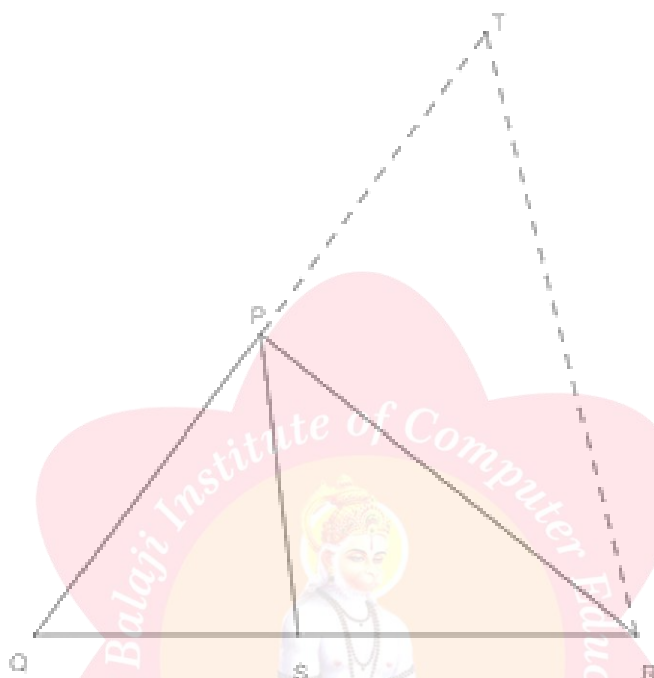




## Chapter - 6: Triangles

### Exercise 6.6 (Page 152 of Grade 10 NCERT)

Q1. In Fig. 6.50, PS is the bisector of  $\angle QPR$  of  $\triangle PQR$ . Prove that  $\frac{QS}{SR} = \frac{PQ}{PR}$



#### Reasoning:

As we know, if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. (BPT)

#### Solution:

Draw a line parallel to PS, through R, which intersects QP produced at T

$$PS \parallel RT$$

In  $\triangle PQR$

$$\angle QPS = \angle SPR \text{ (Since PS is the bisector of } \angle QPR \text{)} \dots\dots\dots (i)$$

$$\text{But } \angle PRT = \angle SPR \text{ (alternate interior angles)} \dots\dots\dots (ii)$$

$$\angle QPS = \angle PTR \text{ (Corresponding angles)} \dots\dots\dots (iii)$$

From (i), (ii), and (iii)

$$\angle PTR = \angle PRT$$

$$PR = PT \dots\dots\dots (iv)$$



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(Since in a triangle sides opposite to equal angles are equal)





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In  $\triangle QRT$ ,  $PS \parallel RT$

$$\frac{QS}{SR} = \frac{QP}{PT} \quad [\text{BPT}]$$

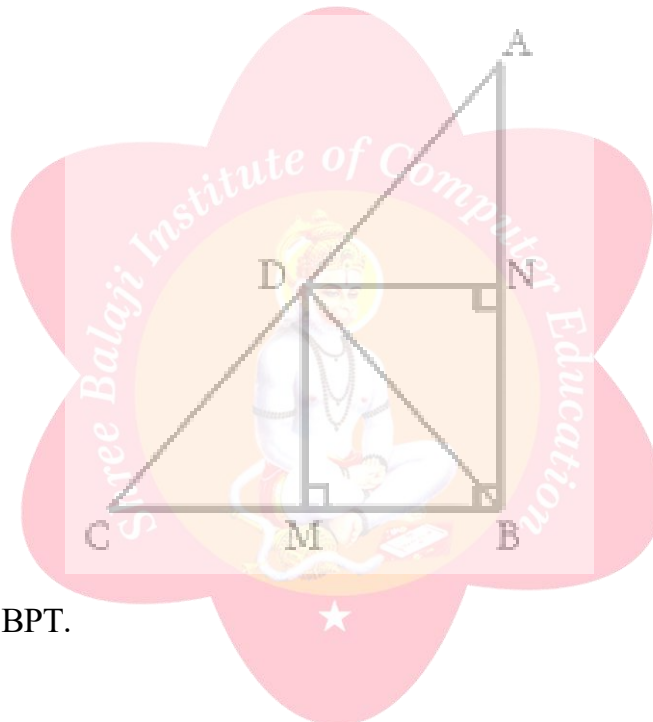
$$\frac{QS}{SR} = \frac{QP}{PR} \quad [\text{from (iv)}]$$

**Q2.** In Fig. 6.57, D is a point on hypotenuse AC of  $\triangle ABC$ , such that  $BD \perp AC$ ,  $DM \perp BC$  and  $DN \perp AB$ .

**Prove that:**

(a)  $DM^2 = DN \cdot MC$

(b)  $DN^2 = DM \cdot AN$



**Reasoning:**

AA similarity criterion, BPT.

**Solution:**

(i) In quadrilateral  $DMBN$

(ii) and

$DMBN$  is a rectangle.

$$DM = BN \text{ and } DN = BM \dots\dots\dots (i)$$

In  $\triangle DCM$

$$\angle DCM + \angle DMC + \angle CDM = 180$$

$$\angle DCM + 90^\circ + \angle CDM = 180$$

$$\angle DCM + \angle CDM = 90^\circ \dots\dots\dots (ii)$$

$$\text{But } \angle CDM + \angle BDM = 90^\circ \dots\dots\dots (iii)$$



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Since  $BD \perp AC$





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From (ii) and (iii)

$$\angle DCM = \angle BDM \dots\dots\dots (iv)$$

In  $\triangle BDM$

$$\angle DBM + \angle BDM = 90 \dots\dots\dots (v)$$

Since,  $DM \perp BC$

From (iii) and (v)

$$\angle CDM = \angle DBM \dots\dots\dots (vi)$$

Now in  $\triangle DCM$  and  $\triangle DBM$

$$\triangle DCM \sim \triangle DBM \text{ (From (iv) and (vi), AA criterion)}$$

$$\frac{DM}{BM} = \frac{MC}{DM} \text{ (Corresponding sides are in same ratio)}$$

$$DM^2 = BM \cdot MC$$

$$DM^2 = DN \cdot MC \quad [\text{from (i) } DN = BM]$$

(iii) In  $\triangle BDN$

$$\angle ABDN + \angle DBN = 90 \text{ (Since } DN \perp AB) \dots\dots\dots (vii)$$

$$\text{But } \angle ADN + \angle ABDN = 90 \text{ (Since } BD \perp AC) \dots\dots\dots (viii)$$

From (vii) and (viii)

$$\angle DBN = \angle ADN \dots\dots\dots (ix)$$

In  $\triangle ADN$

$$\angle DAN + \angle ADN = 90 \text{ (Since } DN \perp AB) \dots\dots\dots (x)$$

$$\text{But } \angle ABDN + \angle ADN = 90 \text{ (Since } BD \perp AC) \dots\dots\dots (xi)$$

From (xi) and (x)

$$\angle DAN = \angle ABDN \dots\dots\dots (xii)$$

Now in  $\triangle BDN$  and  $\triangle DAN$ ,



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$\Delta BDN \sim \Delta DAN$  (From (ix) and (xii), AA criterion)





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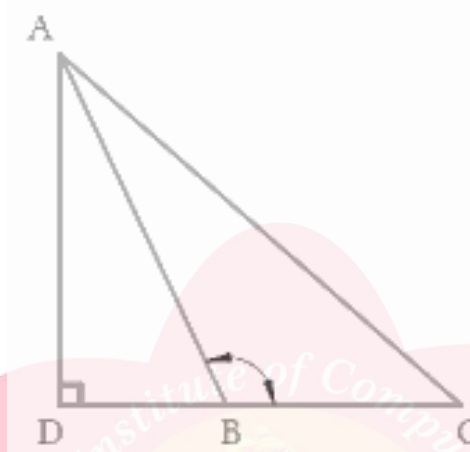
$$\frac{BN}{DN} = \frac{DN}{AN} \quad (\text{Corresponding sides are in same ratio})$$

$$DN^2 = BN \cdot AN$$

$$DN^2 = DM \cdot AN \quad [\text{from (i) } BN = DM]$$

**Q3.** In Fig. 6.58, ABC is a triangle in which  $\angle ABC > 90^\circ$  and  $AD \perp CB$  produced. Prove that:

$$AC^2 = AB^2 + BC^2 + 2BC \cdot BD$$



**Reasoning:**

Pythagoras theorem

**Solution:**

In  $\triangle ADC$

$$\angle ADC = 90^\circ$$

$$\Rightarrow AC^2 = AD^2 + CD^2$$

$$= AD^2 + [BD + BC]^2$$

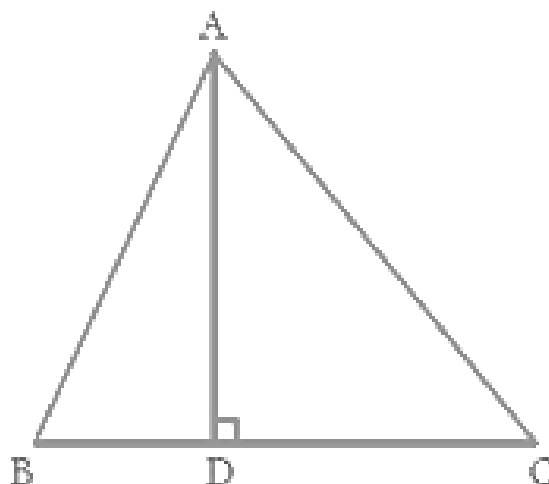
$$= AD^2 + BD^2 + BC^2 + 2BC \cdot BD$$

$$AC^2 = AB^2 + BC^2 + 2BC \cdot BD \quad \left[ \text{In } \triangle ADB, AB^2 = AD^2 + BD^2 \right]$$

**Q4.** In Fig. 6.59, ABC is a triangle in which  $\angle ABC < 90^\circ$  and  $AD \perp BC$ .

**Prove that:**

$$AC^2 = AB^2 + BC^2 - 2BC \cdot BD$$





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## Reasoning:

### Pythagoras Theorem

#### Solution:

In  $\triangle ADC$

$$\angle ADC = 90^\circ$$

$$AC^2 = AD^2 + DC^2$$

$$= AD^2 + [BC - BD]^2$$

$$= AD^2 + BD^2 + BC^2 - 2BC \cdot BD$$

$$AC^2 = AB^2 + BC^2 - 2BC \cdot BD \quad \left[ \text{In } \triangle ADB, AB^2 = AD^2 + BD^2 \right]$$

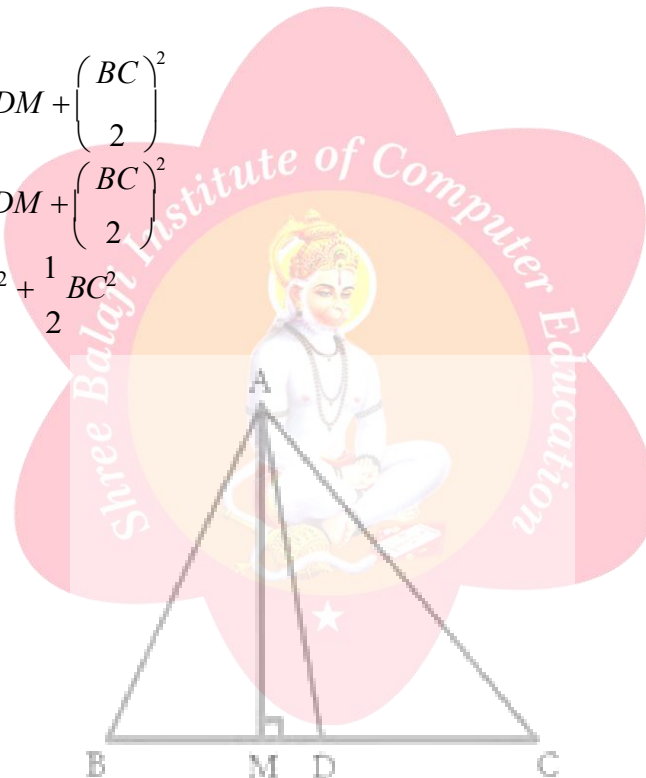
**Q5.** In Fig. 6.60, AD is a median of a triangle ABC and  $AM \perp BC$ .

**Prove that:**

$$\text{i) } AC^2 = AD^2 + BC \cdot DM + \left( \frac{BC}{2} \right)^2$$

$$\text{ii) } AB^2 = AD^2 - BC \cdot DM + \left( \frac{BC}{2} \right)^2$$

$$\text{iii) } AC^2 + AB^2 = 2AD^2 + \frac{1}{2} BC^2$$



## Reasoning:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

#### Solution:

(i) In  $\triangle AMC$

$$\angle AMC = 90^\circ$$

$$AC^2 = AM^2 + CM^2$$

$$= AM^2 + [DM + CD]^2$$

$$= AM^2 + DM^2 + CD^2 + 2DM \cdot CD$$

$$= AD^2 + \left( \frac{BC}{2} \right)^2 + 2DM \left( \frac{BC}{2} \right)$$



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Since, in  $\triangle AMD$ ,  $AD^2 = AM^2 + DM^2$  and D is the midpoint of BC

means  $BD = CD = \frac{BC}{2}$

$$AC^2 = AD^2 + BC \cdot DM + \left(\frac{BC}{2}\right)^2 \dots \dots \dots (i)$$

(ii) In  $\triangle AMB$

$$\angle AMB = 90$$

$$\begin{aligned} AB^2 &= AM^2 + BM^2 \\ &= AM^2 + [BD - DM]^2 \\ &= AM^2 + BD^2 + DM^2 - 2BD \cdot DM \\ &= AM^2 + DM^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right)DM \end{aligned}$$

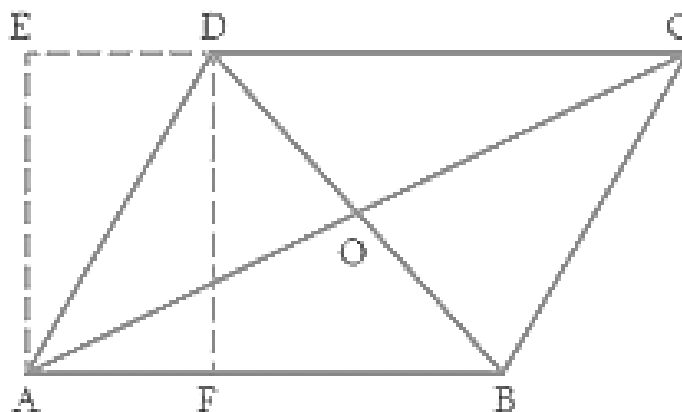
Since, in  $\triangle AMD$ ,  $AD^2 = AM^2 + DM^2$  and D is the midpoint of BC means  $BD = CD = \frac{BC}{2}$

$$AB^2 = AD^2 - BC \cdot DM + \left(\frac{BC}{2}\right)^2 \dots \dots \dots (ii)$$

(iii) Adding (i) and (ii)

$$\begin{aligned} AC^2 + AB^2 &= AD^2 + \left(\frac{BC}{2}\right)^2 + BC \cdot DM + AD^2 + \left(\frac{BC}{2}\right)^2 - BC \cdot DM \\ AC^2 + AB^2 &= 2AD^2 + 2\left(\frac{BC}{2}\right)^2 \\ AC^2 + AB^2 &= 2AD^2 + \frac{1}{2}BC^2 \end{aligned}$$

**Q6.** Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.



## Reasoning:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8



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## Solution:

In parallelogram ABCD

$$AB = CD$$

$$AD = BC$$

Draw  $AE \perp CD$ ,  $DF \perp AB$

$EA = DF$  (Perpendiculars drawn between same parallel lines)

In  $\triangle AEC$

$$\begin{aligned} AC^2 &= AE^2 + EC^2 \\ &= AE^2 + [ED + DC]^2 \\ &= AE^2 + DE^2 + DC^2 + 2DE \cdot DC \\ AC^2 &= AD^2 + DC^2 + 2DE \cdot DC \dots\dots\dots(i) \\ &\quad \left[ \text{Since, } AD^2 = AE^2 + DE^2 \right] \end{aligned}$$

In  $\triangle DFB$

$$\begin{aligned} BD^2 &= DF^2 + BF^2 \\ &= DF^2 + [AB - AF]^2 \\ &= DF^2 + AB^2 + AF^2 - 2AB \cdot AF \\ &= AD^2 + AB^2 - 2AB \cdot AF \\ BD^2 &= AD^2 + AB^2 - 2AB \cdot AF \dots\dots\dots(ii) \\ &\quad \left[ \text{Since, } AD^2 = DF^2 + AF^2 \right] \end{aligned}$$

Adding (i) and (ii)

$$\begin{aligned} AC^2 + BD^2 &= AD^2 + DC^2 + 2DE \cdot DC + AD^2 + AB^2 - 2AB \cdot AF \\ AC^2 + BD^2 &= BC^2 + DC^2 + 2AB \cdot AF + AD^2 + AB^2 - 2AB \cdot AF \end{aligned}$$

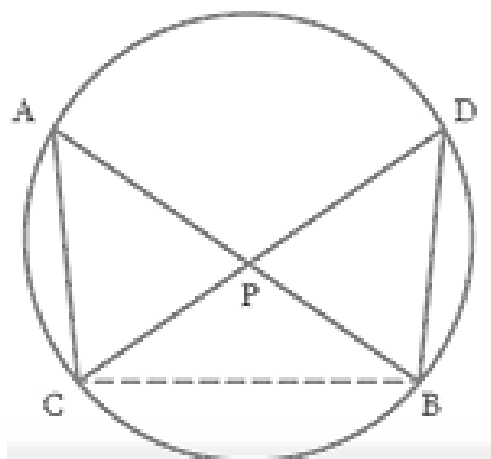
(Since  $AD = BC$  and  $DE = AF$ ,  $CD = AB$ )

$$\Rightarrow AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$$

**Q7.** In Fig. 6.61, two chords AB and CD intersect each other at the point P.

**Prove that:**

- (i)  $\triangle APC \sim \triangle DPB$
- (ii)  $AP \cdot PB = CP \cdot DP$





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## Reasoning:

As we know that, two triangles, are similar if

- (i) Their corresponding angles are equal and
- (ii) Their corresponding sides are in the same ratio

As we know that angles in the same segment of a circle are equal.

## Solution:

Draw BC

- (i) In  $\triangle APC$  and  $\triangle DPB$

$$\angle APC = \angle DPB \quad (\text{Vertically opposite angles})$$

$$\angle PAC = \angle PDB \quad (\text{Angles in the same segment})$$

$$\Rightarrow \triangle APC \sim \triangle DPB \quad (\text{A.A criterion})$$

- (ii) In  $\triangle APC$  and  $\triangle DPB$ ,

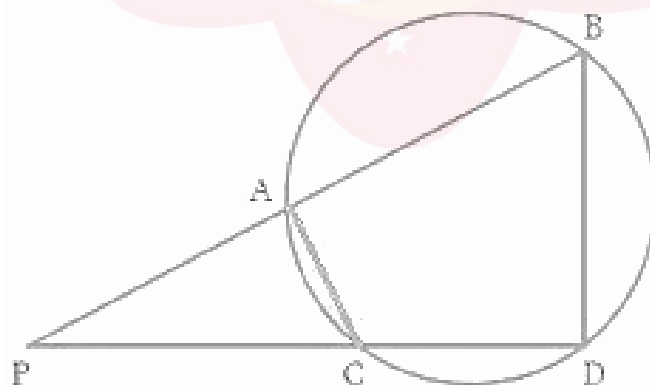
$$\frac{AP}{DP} = \frac{CP}{PB} = \frac{AC}{DB} \quad [\triangle APC \sim \triangle DPB]$$

$$\frac{AP}{DP} = \frac{CP}{PB}$$

$$\Rightarrow AP \cdot PB = CP \cdot DP$$

**Q8.** In Fig. 6.62, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

- (i)  $\triangle PAC \sim \triangle PDB$
- (ii)  $PA \cdot PB = PC \cdot PD$



## Reasoning:

- (i) Exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.
- (ii) Basic proportionality theorem.

## Solution:

Draw AC

- (i) In  $\triangle PAC$  and  $\triangle PDB$

$$\angle APC = \angle BPD \quad (\text{Common angle})$$



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$\angle PAC = \angle PDB$  ( Exterior angle of a cyclic quadrilateral is equal to the opposite interior angle)

$\Rightarrow \triangle PAC \sim \triangle PDB$





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In  $\triangle PAC$  and  $\triangle PDB$

$$\frac{PA}{PD} = \frac{PC}{PB} = \frac{AC}{BD}$$

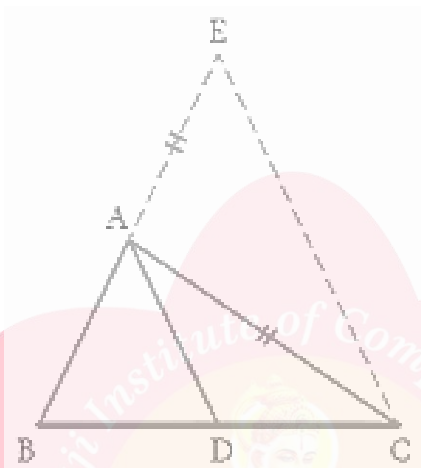
$$\frac{PA}{PD} = \frac{PC}{PB}$$

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$$PA \cdot PB = PC \cdot PD$$

Q9. In Fig. 6.63, D is a point on side BC of  $\triangle ABC$  such that  $\frac{BD}{CD} = \frac{BA}{CA}$ . Prove that AD is the bisector of  $\angle BAC$ .



## Reasoning:

- (i) As we know that in an isosceles triangle, the angles opposite to equal sides are equal.
- (ii) Converse of BPT.

## Solution:

Extended BA to E such that  $AE = AC$  and join CE.

In  $\triangle AEC$

$$AE = AC \Rightarrow \angle ACE = \angle AEC \quad \text{_____ (i)}$$

It is given that

$$\frac{BD}{CD} = \frac{BA}{CA}$$

$$\frac{BD}{CD} = \frac{BA}{CA}$$

$$\frac{BD}{CD} = \frac{BA}{AE} \quad (AC = AE) \quad \text{(ii)}$$

In  $\triangle ABD$  and  $\triangle EBC$

$$AD \parallel EC \text{ (Converse of BPT)}$$

$$\Rightarrow \angle BAD = \angle BEC \text{ (Corresponding angles)} \quad \text{_____ (iii)}$$

$$\text{and } \angle DAC = \angle ACE \text{ (Alternate interior angles)} \quad \text{_____ (iv)}$$

From (i), (iii) and (iv)

$$\angle BAD = \angle DAC$$

$\Rightarrow AD$  is the bisector of



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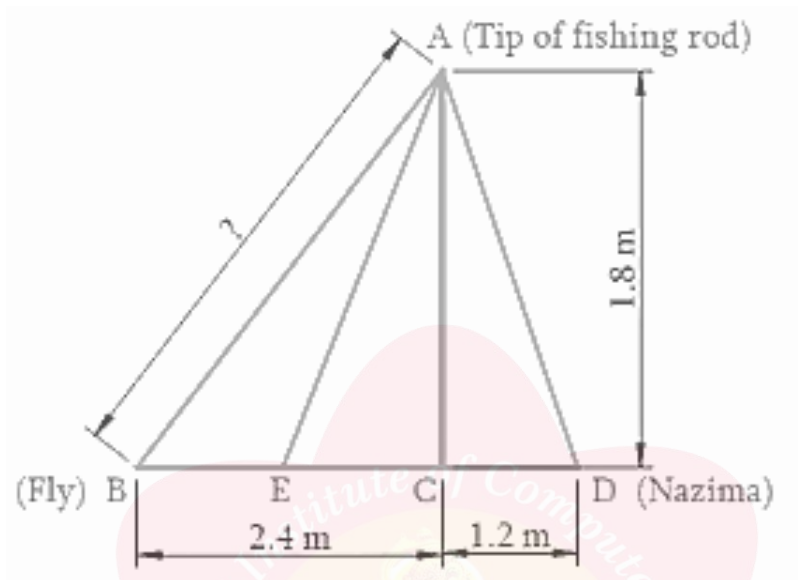
ABAC





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**Q10.** Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Fig. 6.64)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



## Reasoning:

Pythagoras Theorem

## Solution:

To find AB and ED

BD = 3.6 m, BC = 2.4 m, CD = 1.2 m

AC = 1.8 m

In  $\triangle ACB$

$$\begin{aligned} AB^2 &= AC^2 + BC^2 \\ &= (1.8)^2 + (2.4)^2 \\ &= 3.24 + 5.76 \end{aligned}$$

$$AB^2 = 9$$

$$AB = 3$$

Length of the string out AB = 3m

Let the fly at E after 12 seconds

String pulled in 12 seconds =  $12 \times 5$

$$= 60 \text{ cm}$$

$$= 0.6 \text{ m}$$

$$AE = 3\text{m} - 0.6 \text{ m}$$

$$= 2.4 \text{ m}$$



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Now In  $\triangle ACE$

$$\begin{aligned} CE^2 &= AE^2 - AC^2 \\ &= (2.4)^2 - (1.8)^2 \end{aligned}$$

$$\begin{aligned} CE^2 &= 5.76 - 3.24 \\ &= 2.52 \end{aligned}$$

$$CE = 1.587m$$

$$\begin{aligned} DE &= CE + CD \\ &= 1.587 + 1.2 \\ &= 2.787 \end{aligned}$$

$$DE = 2.79 \text{ m}$$

Horizontal distance of the fly after 12 seconds = 2.79 m





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