Shree Balaji Institute

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Chapter 8: Introduction to Trigonometry

Exercise 8.1 (Page 181 of Grade 10 NCERT Textbook)

Q1. In $\Delta ABC,$ right-angled at B, AB = 24 cm, BC = 7 cm, determine:

(i) sin A, cos A

(ii) sin C, cos C

Difficulty level: Easy

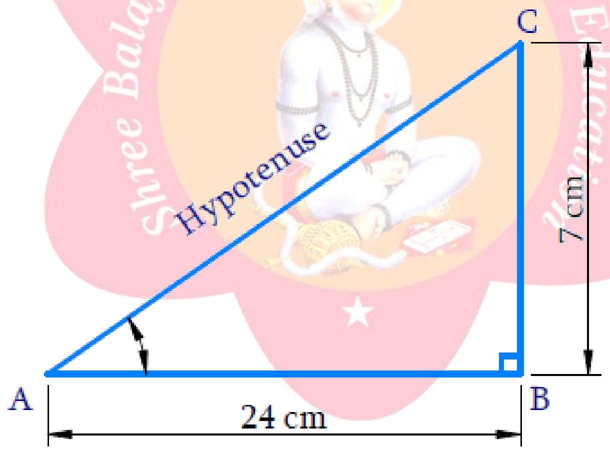
What is the known?

Two sides of a right-angled triangle ΔABC

What is the unknown?

Sine and cosine of angle A and C.

Reasoning:



Applying Pythagoras theorem for $\Delta ABC,$ we can find hypotenuse (side AC). Once hypotenuse is known, we can find sine and cosine angle using trigonometric ratios.

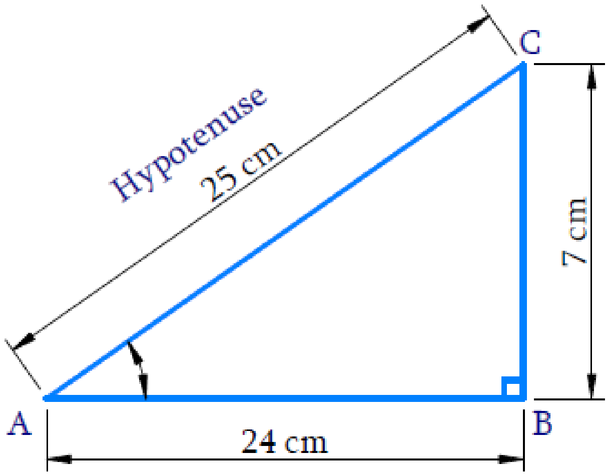
Solution:

In $\Delta ABC,$ we obtain.

$$
\begin{aligned}
AC^2 &= AB^2 + BC^2 \\
&= (24cm)^2 + (7cm)^2 \\
&= (576 + 49)\ cm^2 \\
&= 625\ cm^2
\end{aligned}
$$

$\therefore$ Hypotenuse $AC = \sqrt{625}\ cm = 25\ cm$

(i)



$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\sin A = \frac{7\,\text{cm}}{25\text{cm}} = \frac{7}{25}$$

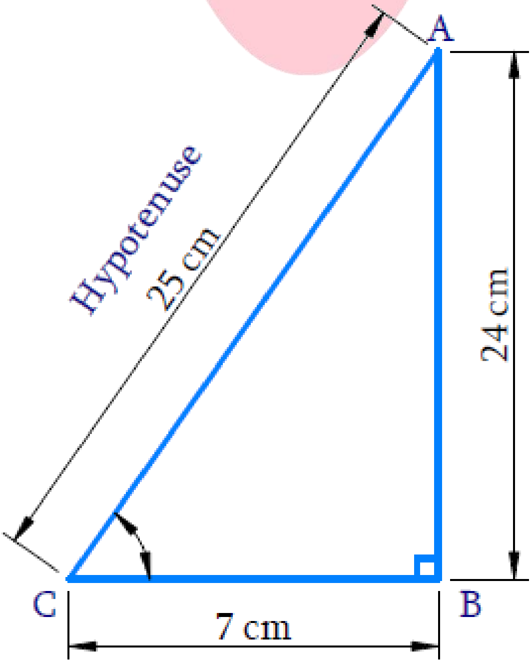
$$\sin A = \frac{7}{25}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{24\text{cm}}{25\text{cm}} = \frac{24}{25}$$

$$\cos A = \frac{24}{25}$$

(ii)



$$\sin C = \frac{\text{side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\sin C = \frac{24\text{cm}}{25\text{cm}} = \frac{24}{25}$$

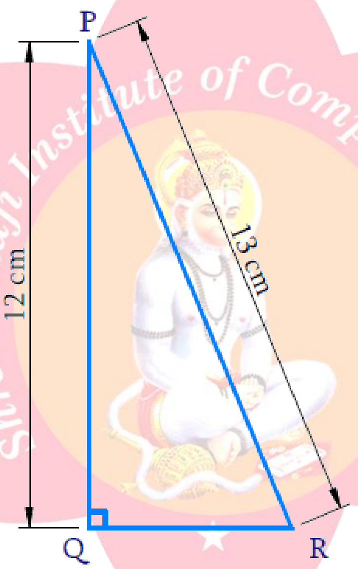
$$\sin C = \frac{24}{25}$$

$$\cos C = \frac{\text{side adjacent to } \angle C}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7\,\text{cm}}{25\text{cm}} = \frac{7}{25}$$

$$\cos C = \frac{7}{25}$$

Q2. In the given figure, find $\tan P - \cot R$.



Difficulty level: Medium

What is the known/given?

PQ = 12 cm and PR = 13 cm.

What is the unknown?

One side of right-angled triangle ΔPQR

Reasoning:

Using Pythagoras theorem, we can find the length of the third side. Then the required trigonometric ratios.

Solution:

Apply Pythagoras theorem for ΔPQR we obtain:

$$PR^2 = PQ^2 + QR^2$$

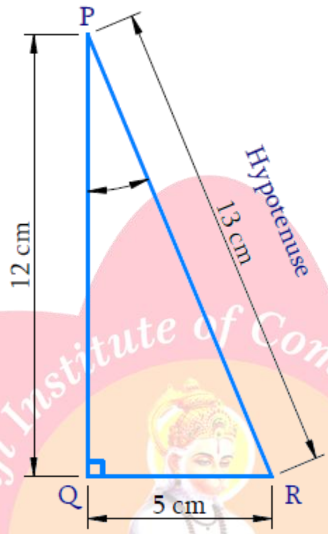
$$QR^2 = PR^2 - PQ^2$$

$$QR^2 = (13cm)^2 - (12cm)^2$$

$$QR^2 = 169cm^2 - 144cm^2$$

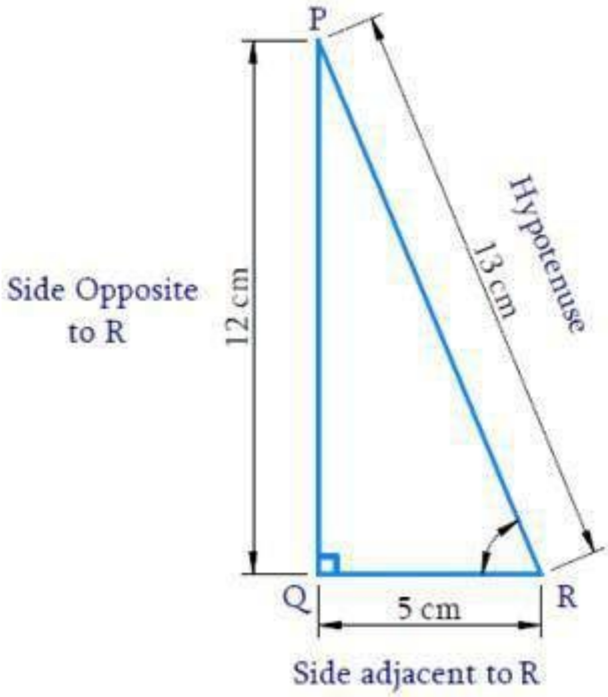
$$QR^2 = 25cm^2$$

$$QR = 5cm$$



$$\tan P = \frac{\text{side opposite to } \angle P}{\text{side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{5cm}{12cm}$$

$$\tan P = \frac{5}{12}$$



$$\cot R = \frac{\text{side adjacent to } \angle R}{\text{side opposite to } \angle R} = \frac{QR}{PQ} = \frac{5cm}{12cm}$$

$$\cot R = \frac{5}{12}$$

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12}$$

$$\tan P - \cot R = 0$$

Q3. If $\sin A = \frac{3}{4}$ calculate cos A and tan A.

Difficulty level: Medium

What is the known/given?

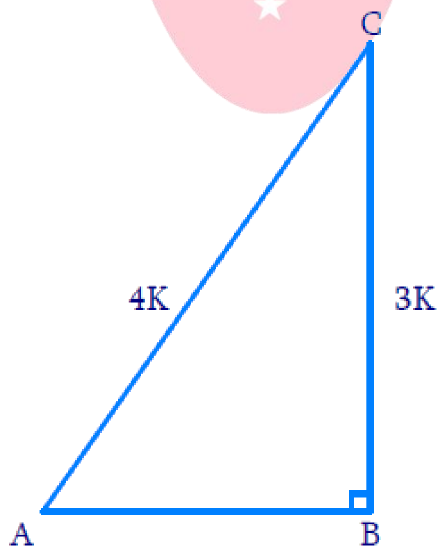
Sine of $\angle A$.

What is the unknown?

Cosine and tangent of $\angle A$

Reasoning:

Using sin A, we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.



Solution:

Let ΔABC be a right-angled triangle, right angled at point B.

Given that

$$\sin A = \frac{3}{4}$$

$$\Rightarrow \frac{BC}{AC} = \frac{3}{4}$$

Let BC be 3k. Therefore, AC will be 4k where k is a positive integer.

Applying Pythagoras theorem for ΔABC, we obtain:

$$AC^2 = AB^2 + BC^2$$

$$AB^2 = AC^2 - BC^2$$

$$AB^2 = (4k)^2 - (3k)^2$$

$$AB^2 = 16k^2 - 9k^2$$

$$AB^2 = 7k^2$$

$$AB = \sqrt{7}k$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k}$$

$$= \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k}$$

$$= \frac{3}{\sqrt{7}}$$

Thus, $\cos A = \frac{\sqrt{7}}{4}$ and $\tan A = \frac{3}{\sqrt{7}}$

Q4. Given 15 cot A = 8, find sin A and sec A.

Difficulty level: Medium

What is the known/given?

Cotangent of $\angle A$

What is the unknown?

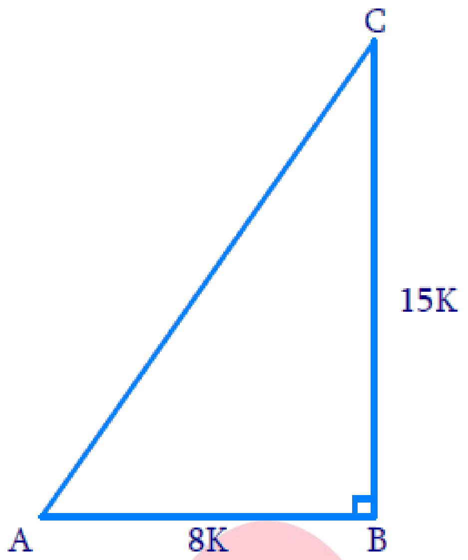
Sine and Secant of $\angle A$.

Reasoning:

Using cot A, we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

Solution:

Let us consider a right-angled ΔABC, right angled at B.



$$\cot A = \frac{\text{side adjacent to } \angle A}{\text{side opposite to } \angle A} = \frac{AB}{BC}$$

It is given that

$$\cot A = \frac{8}{15} \Rightarrow \frac{AB}{BC} = \frac{8}{15}$$

Let AB be 8 k. Therefore, BC will be 15 k where k is a positive integer.

Apply Pythagoras theorem in ΔABC, we obtain.

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = (8k)^2 + (15k)^2$$
$$AC^2 = 64k^2 + 225k^2$$
$$AC^2 = 289k^2$$
$$AC = 17k$$

$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{15k}{17k}$$
$$= \frac{15}{17}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle A} = \frac{AC}{AB} = \frac{17k}{8k}$$
$$= \frac{17}{8}$$

Thus, $\sin A = \frac{15}{17}$ and $\sec A = \frac{17}{8}$

Q5. Given $\sec 8 = \frac{13}{12}$, calculate all other trigonometric ratios.

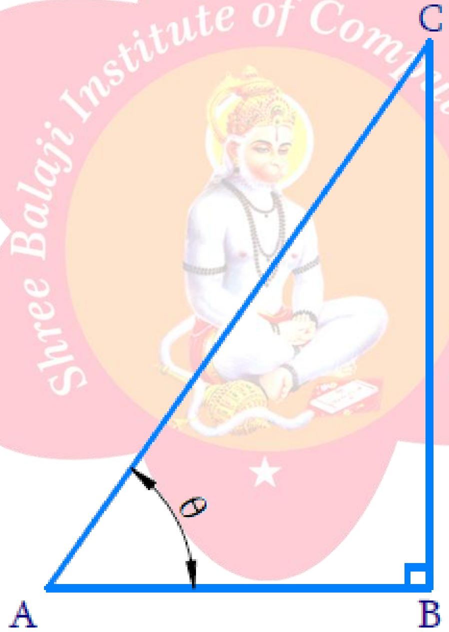
Difficulty level: Medium

What is the known/given?
Secant of 8

What is the unknown?
Other trigonometric ratios.

Reasoning:
Using $\text{Sec}\,\theta$, we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

Solution:
Let ΔABC be a right-angled triangle, right angled at point B.



It is given that:

$$\sec\theta = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle 8} = \frac{AC}{AB} = \frac{13}{12}$$

Let AC = 13 k and AB = 12 k where k is a positive integer.

Apply Pythagoras theorem in $\Delta\,ABC$, we obtain:

$$AC^2 = AB^2 + BC^2$$
$$BC^2 = AC^2 - AB^2$$
$$BC^2 = (13k)^2 - (12k)^2$$
$$BC^2 = 169k^2 - 144k^2$$
$$BC^2 = 25k^2$$
$$BC = 5k$$

$$\sin\theta = \frac{\text{side opposite to } \angle\theta}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{5}{13}$$

$$\cos\theta = \frac{\text{side adjacent to } \angle\theta}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{12}{13}$$

$$\tan\theta = \frac{\text{side opposite to } \angle\theta}{\text{side adjacent to } \angle\theta} = \frac{BC}{AB} = \frac{5}{12}$$

$$\cot\theta = \frac{\text{side adjacent to } \angle\theta}{\text{side opposite to } \angle\theta} = \frac{AB}{BC} = \frac{12}{5}$$

$$\operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{side opposite to } \angle\theta} = \frac{AC}{BC} = \frac{13}{5}$$

Q6. If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Difficulty level: Medium

What is the known/given?

$\angle A$ and $\angle B$ are acute angles and $\cos A = \cos B$.

What is the unknown?

To show that $\angle A = \angle B$

Reasoning:

Using cos A and cos B, we can find the ratio of the length of two sides of the right-angled triangle with respective angles. Then compare both the ratios.

Solution:

In the right-angled triangle ABC, ∠A and ∠B are acute angles and ∠C is right angle.

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AC}{AB}$$

$$\cos B = \frac{\text{side adjacent to} \angle B}{\text{hypotenuse}} = \frac{BC}{AB}$$

Given that $\cos A = \cos B$

Therefore,

$$\frac{AC}{AB} = \frac{BC}{AB}$$

$$AC = BC$$

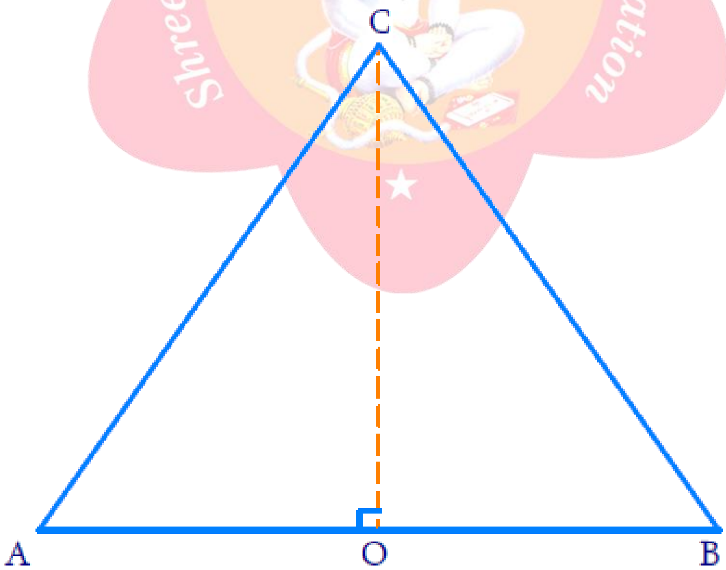
Hence, $\angle A = \angle B$, (angles opposite to equal sides of triangle are equal.)

Alternatively,

Reasoning:

Using cos A and cos B, we can find the ratio of the length of two sides of the right-angled triangle with respective angles. Then by using Pythagoras theorem, relation between the sides.

Let us consider a triangle ABC in which CO ⊥ AB.



It is given that

$$\cos A = \cos B$$

$$\frac{AO}{AC} = \frac{BO}{BC}$$

$$\frac{AO}{BO} = \frac{AC}{BC}$$

Let $\frac{AO}{BO}=\frac{AC}{BC}=k$

$AO=k.BO$ (i)

$AC=k.BC$ (ii)

By applying Pythagoras theorem in ΔCAO and ΔCBO, we get.

$AC^2=AO^2+CO^2$ from ΔCAO

$CO^2=AC^2-AO^2$ (iii)

$BC^2=BD^2+CO^2$ from ΔCBO

$CO^2=BC^2-BO^2$ (iv)

From equation (iii) and equation (iv), we get

$$AC^2-AO^2=BC^2-BO^2$$

$$(kBC)^2-(kBO)^2=BC^2-BO^2$$

$$k^2BC^2-k^2BO^2=BC^2-BO^2$$

$$k^2(BC^2-BO^2)=BC^2-BO^2$$

$$k^2=\frac{BC^2-BO^2}{BC^2-BO^2}=1$$

$$k=1$$

Putting this value in equation (ii) we obtain

$AC=BC$

$\angle A=\angle B$ (angles opposite to equal sides of triangle are equal.)

Q7. If $\cot 8 = \frac{7}{8}$, evaluate: (i) $\frac{(1+\sin 8)(1-\sin 8)}{(1+\cos 8)(1-\cos 8)}$, (ii) $\cot^2 8$

Difficulty Level: Medium

What is the known/given?

$\cot 8 = \frac{7}{8}$

What is the unknown?

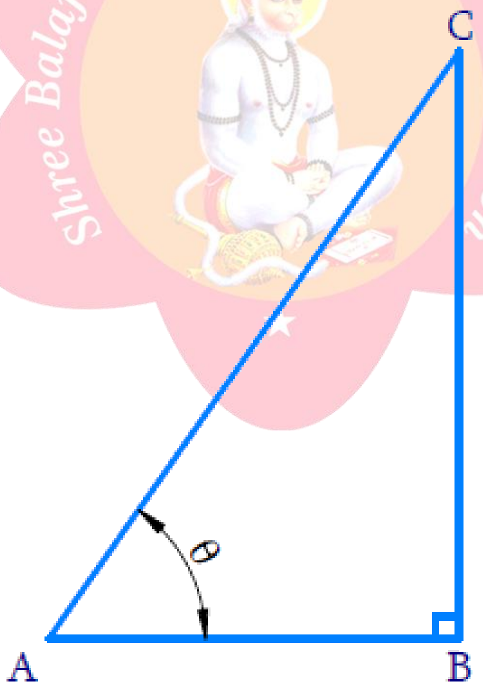
Value of (i) $\frac{(1+\sin 8)(1-\sin 8)}{(1+\cos 8)(1-\cos 8)}$, and (ii) $\cot^2 8$

Reasoning:

Using $\cot 8 = \frac{7}{8}$, we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

Solution:

Let ΔABC, in which angle B is right angle.



$$\cot 8 = \frac{\text{side adjacent to } 8}{\text{side opposite to } 8} = \frac{AB}{BC} = \frac{7}{8}$$

Let $AB = 7k$ and $BC = 8k$, where k is a positive integer.

By applying Pythagoras theorem in ΔABC, we get.

$$AC^2 = AB^2 + BC^2$$
$$= (7k)^2 + (8k)^2$$
$$= 49k^2 + 64k^2$$
$$= 113k^2$$
$$AC = \sqrt{113k^2}$$
$$= \sqrt{113}k$$

Therefore,

$$\sin 8 = \frac{\text{side opposite to } 8}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos 8 = \frac{\text{side adjacent to } 8}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i) $\dfrac{(1+\sin 8)(1-\sin 8)}{(1+\cos 8)(1-\cos 8)}$

$$\frac{(1+\sin 8)(1-\sin 8)}{(1+\cos 8)(1-\cos 8)} = \frac{1-\sin^2 8}{1-\cos^2 8} \quad \left[\because (a+b)(a-b) = (a^2-b^2)\right]$$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2}$$

$$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{49/113}{64/113}$$

$$= \frac{49}{64}$$

(ii) $\cot^2 8$

$$\cot^2 8 = \left(\frac{7}{8}\right)^2$$

$$= \frac{49}{64}$$

Q8. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A}=\cos^2 A-\sin^2 A$ or not.

Difficulty Level: Medium

What is the known/given?
Cotangent of angle A

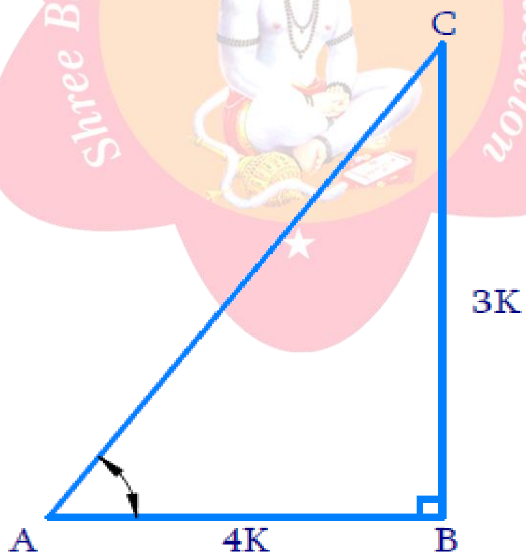
What is the unknown?
whether $\frac{1-\tan^2 A}{1+\tan^2 A}=\cos^2 A-\sin^2 A$

Reasoning:
Using $3\cot A=4$, we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

Solution:

$$3\cot A = 4$$
$$\cot A = \frac{4}{3}$$

Let ΔABC, in which angle B is right angle.



$$\cot A=\frac{\text{side adjacent to }\angle A}{\text{side opposite to }\angle A}=\frac{AB}{BC}=\frac{4}{3}$$

Let $AB=4k$ and $BC=3k$ where k is a positive integer.

By applying Pythagoras theorem in ΔABC, we get.

$$AC^2 = AB^2 + BC^2$$
$$= (4k)^2 + (3k)^2$$
$$= 16k^2 + 9k^2$$
$$= 25k^2$$
$$AC = \sqrt{25k^2}$$
$$= 5k$$

Therefore,

$$\tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$$
$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$$
$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$L.H.S = \frac{1-\tan^2 A}{1+\tan^2 A}$$
$$= \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$$
$$= \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$
$$= \frac{16-9}{16+9}$$
$$= \frac{7}{25}$$

$$R.H.S = \cos^2 A - \sin^2 A$$
$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$
$$= \frac{16}{25} - \frac{9}{25}$$
$$= \frac{16-9}{25}$$
$$= \frac{7}{25}$$

Therefore, $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

Q9. In the triangle ABC right-angled at B, if $\tan A = \dfrac{1}{\sqrt{3}}$ find the value of:

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

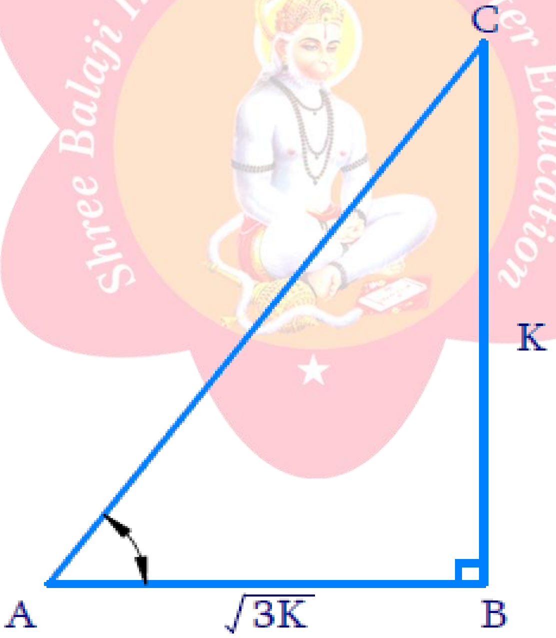
Difficulty level: Medium

Reasoning:

Using $\tan A = \dfrac{1}{\sqrt{3}}$, we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

Solution:

(i) Let ΔABC be a right-angled triangle $\tan A = \dfrac{1}{\sqrt{3}}$



$$\tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

Let $BC = k \text{ and } AB = \sqrt{3}k$ where k is a positive real number.

By applying Pythagoras theorem for ΔABC

$$AC^2 = AB^2 + BC^2$$
$$= \left(\sqrt{3}k\right) + (k)^2$$
$$= 3k^2 + k^2$$
$$= 4k^2$$
$$AC = \sqrt{4k^2}$$
$$= 2k$$

Therefore,

$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{1}{2}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{side adjacent to } \angle C}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{1}{2}$$

(i) $\sin A \cos C + \cos A \sin C$

By substituting the values of the trigonometric functions in the above equation.

$$\sin A \cos C + \cos A \sin C == \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{1}{4} + \frac{3}{4}$$
$$= \frac{1+3}{4}$$
$$= \frac{4}{4}$$
$$= 1$$

(ii) $\cos A \cos C - \sin A \sin C$

By substituting the values of the trigonometric functions in the above equation.

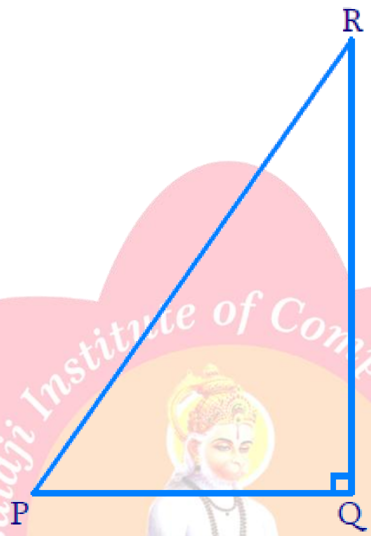
$$\cos A \cos C - \sin A \sin C = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$
$$= 0$$

Q10. In ΔPQR, right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of sin P, cos P and tan P.

Difficulty level: Medium

Reasoning:
Using Pythagoras theorem, we can find the length of the all three sides. Then the required trigonometric ratios

Solution:
Given, Δ PQR is right-angled at Q.



$$PQ = 5cm$$
$$PR + QR = 25cm$$

Let $PR = x\ cm$

Therefore,

$$QR = 25\text{cm} - PR$$
$$= (25 - x)\text{cm}$$

By applying Pythagoras theorem for Δ PQR, we obtain.

$$PR^2 = PQ^2 + QR^2$$
$$x^2 = (5)^2 + (25 - x)^2$$
$$x^2 = 25 + 625 - 50x + x^2$$
$$50x = 650$$
$$x = \frac{650}{50}$$
$$= 13$$

Therefore,

$$PR = 13cm$$
$$QR = (25-13)\ cm$$
$$= 12\ cm$$

By substituting the values obtained above in the trigonometric functions below.

$$\sin P = \frac{\text{side opposite to } \angle P}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$
$$\cos P = \frac{\text{side adjacent to } \angle P}{\text{hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$
$$\tan P = \frac{\text{side opposite to } \angle P}{\text{side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

Q11. State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A.

(v) $\sin 8 = \frac{4}{3}$, for some angle θ.

Difficulty level: Medium

Solution:

(i) False, because sides of a right-angled triangle may have any length. So tan A may have any value.

(ii)

$$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle A}$$

As hypotenuse is the largest side, the ratio on RHS will be greater than 1. Hence $\sec A > 1$. Thus, the given statement is true.

(iii) Abbreviation used for cosecant of $\angle A$ is cosec A and cos A is the abbreviation used for cosine of $\angle A$. Hence the given statement is false.

(iv) cot A is not the product of cot and A. It is the cotangent of $\angle A$. Hence, the given statement is false.

(v) $\text{Sin}\,8 = \frac{4}{3}$

We know that in a right-angled triangle,

$$\text{Sin}\,8 = \frac{\text{side adjacent to } \angle 8}{\text{hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Also, the value of Sine should be less than 1. Therefore, such value of Sin8 is not possible. Hence the given statement is false.

Chapter 8: Introduction to Trigonometry

Exercise 8.2 (Page 187 of Grade 10 NCERT Textbook)

Q1. Evaluate the following:

(i) $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0$

$2\tan^2 45^0 + \cos^2 30^0 - \sin^2 60^0$

(iii) $\dfrac{\cos 45^0}{\sec 30^0 + \operatorname{cosec} 30^0}$

(iv) $\dfrac{\sin 30^0 + \tan 45^0 - \operatorname{cosec} 60^0}{\sec 30^0 + \cos 60^0 - \cot 45^0}$

(v) $\dfrac{5\cos^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\sec^2 30^0 + \cos^2 30^0}$

Difficulty level: Medium

Reasoning:

We know that,

Exact Values of Trigonometric Functions				
Angle (θ)		sin (θ)	cos (θ)	tan (θ)
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Not Defined

Solution:

(i)

$$\begin{aligned}\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0 &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{3}{4} + \frac{1}{4} \\ &= \frac{3+1}{4} \\ &= \frac{4}{4} \\ &= 1\end{aligned}$$

(ii)

$$2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ = 2\left(\tan 45^\circ\right)^2 + \left(\cos 30^\circ\right)^2 - \left(\sin 60^\circ\right)^2$$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4}$$

$$= 2$$

(iii)

$$\frac{\cos 45^0}{\sec 30^0 + \operatorname{cosec} 30^0} = \frac{\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{2}{\sqrt{3}}\right) + \left(\frac{2}{1}\right)}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2 + 2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{1 \times \sqrt{3}}{\sqrt{2} \times \left(2 + 2\sqrt{3}\right)}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}\left(\sqrt{3} + 1\right)}$$

Multiplying numerator and denominator by $\sqrt{2}\left(\sqrt{3} - 1\right)$, we get

$$= \frac{\sqrt{3}}{2\sqrt{2}\left(\sqrt{3} + 1\right)} \times \frac{\sqrt{2}(\sqrt{3} - 1)}{\sqrt{2}(\sqrt{3} - 1)}$$

$$= \frac{3\sqrt{2} - \sqrt{6}}{4\left(3 - 1\right)}$$

$$= \frac{3\sqrt{2} - \sqrt{6}}{8}$$

(iv)

$$\frac{\sin 30^0 + \tan 45^0 - \operatorname{cosec} 60^0}{\sec 30^0 + \cos 60^0 + \cot 45^0} = \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$= \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{3}{2}}$$

$$= \frac{\frac{3\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + 3\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4}$$

Multiplying numerator and denominator by $\left(3\sqrt{3} - 4\right)$, we get

$$= \frac{(3\sqrt{3} - 4)\,(3\sqrt{3} - 4)}{(3\sqrt{3} + 4)\,(3\sqrt{3} - 4)}$$

$$= \frac{27 + 16 - 24\sqrt{3}}{27 - 16}$$

$$= \frac{43 - 24\sqrt{3}}{11}$$

(v)

$$\frac{5\cos^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\sin^2 30^0 + \cos^2 30^0} = \frac{5\times\left(\frac{1}{2}\right)^2 + 4\times\left(\frac{2}{\sqrt{3}}\right)^2 - (-1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{\left(\frac{5}{4} + \frac{16}{3} - 1\right)}{\left(\frac{1}{4} + \frac{3}{4}\right)}$$

$$= \frac{\left(\frac{15 + 64 - 12}{12}\right)}{\left(\frac{3+1}{4}\right)}$$

$$= \frac{\left(\frac{67}{12}\right)}{\left(\frac{4}{4}\right)}$$

$$= \frac{67}{12}$$

Q2. Choose the correct option and justify your choice:

(i) $\dfrac{2\tan 30^0}{1 + \tan^2 30^0}$

(A) $\sin 60^0$ (B) $\cos 60^0$ (C) $\tan 60^0$ (D) $\sin 60^0$

(ii) $\dfrac{1 - \tan^2 45^0}{1 + \tan^2 45^0}$

(A) $\tan 90^0$ (B) 1 (C) $\sin 45^0$ (D) 0^0

(iii) $\sin 2A = 2\sin A$ is true when $A =$

(A) 0^0 (B) 30^0 (C) 45^0 (D) 60^0

(iv) $\dfrac{2\tan 30^0}{1-\tan^2 30^0}$

(A) $\cos 60^0$ (B) $\sin 60^0$ (C) $\tan 60^0$ (D) $\sin 30^0$

Difficulty level: Medium

Reasoning:

We know that,

Exact Values of Trigonometric Functions				
Angle (θ)		sin (θ)	cos (θ)	tan (θ)
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Not Defined

Solution:

(i) $\dfrac{2\tan 30^0}{1+\tan^2 30^0}$

By substituting the values of given trigonometric ratios in the above equation, we get.

$$=\frac{2\times\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$=\frac{2\times\frac{1}{\sqrt{3}}}{1+\frac{1}{3}}$$

$$=\frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$=\frac{2}{\sqrt{3}}\times\frac{3}{4}$$

$$=\frac{\sqrt{3}}{2}$$

Out of the given options only $\sin 60^0 = \frac{\sqrt{3}}{2}$. Hence, option (A) is correct.

(ii)

$$\frac{1-\tan^2 45^0}{1+\tan^2 45^0}$$

By substituting the values of given trigonometric ratios for $\tan 45^0$.

$$= \frac{1-(1)^2}{1+(1)^2}$$

$$= \frac{1-1}{1+1}$$

$$= \frac{0}{2}$$

$$= 0$$

Hence, option (D) is correct.

(iii)

$$\sin 2A = 2\sin A$$

By substituting $A = 0^0, 30^0, 45^0$ and 60^0 ,we get

For $A = 0^0$

$$\sin 2A = \sin 2\times 0^0$$
$$= \sin 0^0$$
$$= 0$$
$$2\sin A = 2\times\sin 0^0$$
$$= 2\times 0^0$$
$$= 0$$
$$\sin 2A = 2\sin A \qquad \text{(When } A = 0^0\text{)}$$

For $A = 30^0$

$$\sin 2A = \sin 2\times 30^0$$
$$= \sin 60^0$$
$$= \frac{\sqrt{3}}{2}$$
$$2\sin A = 2\times\sin 30^0$$
$$= 2\times\frac{1}{2}$$
$$= 1$$
$$\sin 2A \neq 2\sin A \qquad \text{(When } A = 30^0\text{)}$$

For $A = 45^0$

$$\sin 2A = \sin 2\times 45^0$$
$$= \sin 90^0$$
$$= 1$$
$$2\sin A = 2\times \sin 45^0$$
$$= 2\times \frac{1}{\sqrt{2}}$$
$$= \sqrt{2}$$
$$\sin 2A \neq 2\sin A \qquad (\text{When } A = 45^0)$$

For $A = 60^0$

$$\sin 2A = \sin 2\times 60^0$$
$$= \sin 120^0$$
$$= \frac{\sqrt{3}}{2}$$
$$2\sin A = 2\times \sin 60^0$$
$$= 2\times \frac{\sqrt{3}}{2} = \sqrt{3}$$
$$\sin 2A \neq 2\sin A \qquad (\text{When } A = 60^0)$$

Hence Option (A) is correct

(iv)

$$\frac{2\tan 30^0}{1-\tan^2 30^0}$$

By substituting the values of given trigonometric ratios for $\tan 30^0$, we get

$$= \frac{2\times\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$$
$$= \frac{\left(\frac{2}{\sqrt{3}}\right)}{\left(1-\frac{1}{3}\right)}$$
$$= \frac{\left(\frac{2}{\sqrt{3}}\right)}{\left(\frac{2}{3}\right)}$$
$$= \frac{2}{\sqrt{3}}\times\frac{3}{2}$$
$$= \sqrt{3}$$

Out of the given option only $\tan 60^0 = \sqrt{3}$.

Hence option (C) is correct.

Q3. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$; $0^0 < (A+B) \le 90^0$, $A > B$, find A and B.

Difficulty level: Medium

Solution:

Given that

$$\tan(A+B) = \sqrt{3} \text{ and, } \tan(A-B) = \frac{1}{\sqrt{3}}$$

$$\text{Since, } \tan 60^0 = \sqrt{3} \text{ and } \tan 30^0 = \frac{1}{\sqrt{3}}$$

Therefore,

$$\tan(A+B) = \tan 60^0$$

$$(A+B) = 60^0 \quad \text{(i)}$$

$$\tan(A-B) = \tan 30^0$$

$$(A-B) = 30^0 \quad \text{(ii)}$$

On adding both equations (i) and (ii), we obtain:

$$A+B+A-B = 60^0 + 30^0$$

$$2A = 90^0$$

$$A = 45^0$$

By substituting the value of A in equation (i) we obtain

$$A+B = 60^0$$

$$45^0 + B = 60^0$$

$$B = 60^0 - 45^0 = 15^0$$

Therefore, $\angle A = 45^0$ and $\angle B = 15^0$ $(A > B)$

Q4. State whether the following are true or false. Justify your answer.

(i) $\sin(A+B) = \sin A + \sin B$.

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) The value of $\cos\theta$ increases as θ increases.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

(v) $\cot A$ is not defined for $A = 0°$.

Difficulty level: Medium

Solution:

$$\sin(A+B)=\sin A+\sin B.$$

For the purpose of verification, Let $A=30^0$ and $B=60^0$

$$\begin{aligned}\text{L.H.S} &= \sin(A+B)\\ &=\sin(30^0+60^0)\\ &=\sin 90^0\\ &=1\end{aligned}$$

$$\begin{aligned}\text{R.H.S} &= \sin A+\sin B\\ &=\sin 30^0+\sin 60^0\\ &=\frac{1}{2}+\frac{\sqrt{3}}{2}\\ &=\frac{1+\sqrt{3}}{2}\end{aligned}$$

Since, $\sin(A+B)\neq\sin A+\sin B.$

Hence, the given statement is not true

(ii) The value of $\sin 8$ increases from 0 to 1 as θ increases from 0^0 to 90^0

$$\sin 0^0=0$$

$$\sin 30^0=\frac{1}{2}=0.5$$

$$\sin 45^0=\frac{1}{\sqrt{2}}=0.707$$

$$\sin 60^0=\frac{\sqrt{3}}{2}=0.866$$

$$\sin 90^0=1$$

Hence, the given statement is true.

(iii) The value of $\cos 8$ decreases from 1 to 0 as θ increases from 0^0 to 90^0

$$\cos 0^0=1$$

$$\cos 30^0=\frac{\sqrt{3}}{2}=0.866$$

$$\cos 45^0=\frac{1}{\sqrt{2}}=0.707$$

$$\cos 60^0=\frac{1}{2}=0.5$$

$$\cos 90^0=0$$

Hence, the given statement is false.

(iv)

This is true when $8 = 45^0$

As $\sin 45^0 = \frac{1}{\sqrt{2}}$ and $\cos 45^0 = \frac{1}{\sqrt{2}}$

It is not true for other values of θ

As,

$$\sin 30^0 = \frac{1}{2} \text{ and } \cos 30^0 = \frac{\sqrt{3}}{2}$$

$$\sin 60^0 = \frac{\sqrt{3}}{2} \text{ and } \cos 60^0 = \frac{1}{\sqrt{2}}$$

$$\sin 90^0 = 1 \text{ and } \cos 90^0 = 0$$

Hence, the given statement is false.

(v)

$$\cot A = \frac{\cos A}{\sin A}$$

$$\therefore \cot 0^0 = \frac{\cos 0^0}{\sin 0^0} = \frac{1}{0} = \text{ undefined}$$

Hence the given statement is true.

Chapter 8: Introduction to Trigonometry

Exercise 8.3 (Page 189 of Grade 10 NCERT Textbook)

Q1. Evaluate:

(i) $\dfrac{\sin 18^0}{\cos 72^0}$ (ii) $\dfrac{\tan 26^0}{\cot 64^0}$ (iii) $\cos 48^0 - \sin 42^0$ (iv) $\text{cosec} 31^0 - \sec 59^0$

Difficulty level: Medium

Reasoning:

$$\sin\left(90^0 - \theta\right) = \cos\theta$$
$$\tan(90^0 - 8) = \cot 8$$
$$\sec(90^0 - 8) = \text{cosec} 8$$

Solution:

(i)

$$\frac{\sin 18^0}{\cos 72^0}$$

Since,

$$\sin\left(90^0 - 8\right) = \cos 8$$

Here $\theta = 72^0$

$$= \frac{\sin(90^0 - 72^0)}{\cos 72^0}$$
$$= \frac{\cos 72^0}{\cos 72^0}$$
$$= 1$$

(ii)

$$\frac{\tan 26^0}{\cot 64^0}$$

Since

$$\tan(90^0 - 8) = \cot 8$$

Here $8 = 64^0$

$$=\frac{\tan(90^0-64^0)}{\cot 64^0}$$
$$=\frac{\cot 64^0}{\cot 64^0}$$
$$=1$$

(iii)

$$\cos 48^0 - \sin 42^0$$

Since,

$$\sin(90^0-\theta)=\cos\theta$$

Here $\theta=48^0$

$$=\cos 48^0-\sin\left(90^0-48^0\right)$$
$$=\cos 48^0-\cos 48^0$$
$$=0$$

(iv)

$$\text{cosec}31^0 - \sec 59^0$$

Since,

$$\sec(90^0-8)=\text{cosec}8$$

Here $8=31^0$

$$=\text{cosec}31^0-\sec\left(90^0-31^0\right)$$
$$=\text{cosec}31^0-\text{cosec}31^0$$
$$=0$$

Q2. Show that:

(i) $\tan 48°\tan 23°\tan 42°\tan 67°=1$

(ii) $\cos 38°\cos 52°-\sin 38°\sin 52°=0$

Difficulty level: Medium

Reasoning:

$$\sin\left(90^0-\theta\right)=\cos\theta$$
$$\tan(90^0-8)=\cot 8$$

Solution:

(i) Taking L.H.S

$$= \tan 48° \tan 23° \tan 42° \tan 67°$$

Since $\tan(90^0 - \theta) = \cot\theta$

$$= \tan(90^0 - 42^0)\tan(90^0 - 67^0)\tan 42^0 \tan 67^0$$

$$= \cot 42^0 \cot 67^0 \tan 42^0 \tan 67^0$$

$$= (\cot 42^0 \tan 42^0)(\cot 67^0 \tan 67^0)$$

$$= \left(\frac{1}{\tan 42^0} \times \tan 42^0\right)\left(\frac{1}{\tan 67^0} \times \tan 67^0\right)$$

$$= 1 \times 1$$

$$= 1$$

$$= \text{R.H.S}$$

Hence, $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

(ii) Taking L.H.S

$$= \cos 38° \cos 52° - \sin 38° \sin 52°$$

Since, $\sin(90^0 - \theta) = \cos\theta$

$$= \cos 38^0 \cos 52^0 - \sin(90^0 - 57^0)\sin(90^0 - 38^0)$$

$$= \cos 38^0 \cos 52^0 - \cos 52^0 \cos 38^0$$

$$= 0$$

$$= \text{R.H.S}$$

Hence, $\cos 38^0 \cos 52^0 - \sin 38^0 \sin 52^0 = 0$

Q3. If $\tan 2A = \cot(A - 18°)$, where *2A* is an acute angle, find the value of A.

Difficulty level: Medium

Reasoning:

$$\tan(90^0 - 8) = \cot 8$$

Solution:

Given that: $\tan 2A = \cot(A - 18°)$ ….(i)

But $\tan 2A = \cot(90^0 - 2A)$

By substituting this in equation (i) we get:

$$\cot\left(90^0-2A\right)=\cot\left(A-18^0\right)$$
$$90^0-2A=A-18^0$$
$$3A=108^0$$
$$A=\frac{108^0}{3}=$$
$$A=36^0$$

Q4. If $\tan A=\cot B,$ prove that $A+B=90^0.$

Difficulty level: Easy

Reasoning:

$$\tan(90^0-8)=\cot 8$$

Solution:

Given that: $\tan A=\cot B$ (i)

We know that $\tan A=\cot\left(90^0-A\right)$

By substituting this in equation (i) we get:

$$\cot(90^0-A)=\cot B$$
$$90^0-A=B$$
$$A+B=90^0$$

Q5. If $\sec 4A=\operatorname{cosec}\left(A-20^\circ\right),$ where 4A is an acute angle, find the value of A.

Difficulty level: Easy

Reasoning:

$$\sec A=\operatorname{cosec}\left(90^0-A\right)$$

Solution:

Given that: $\sec 4A=\operatorname{cosec}\left(A-20^\circ\right)$....(i)

Since, $\sec A=\operatorname{cosec}\left(90^0-A\right)$

By using property in equation (i) we get:

$$\operatorname{cosec}\left(90^0 - 4A\right) = \operatorname{cosec}\left(A - 20^0\right)$$
$$90^0 - 4A = A - 20^0$$
$$5A = 110^0$$
$$A = \frac{110^0}{5}$$
$$A = 22^0$$

Q6. If A, B and C are interior angles of a triangle ABC, then show that $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

Difficulty level: Medium

Reasoning:

$$\sin\left(90^0 - \theta\right) = \cos\theta$$

Solution:

We know that for ΔABC,

$$\angle A + \angle B + \angle C = 180^0$$
$$\angle B + \angle C = 180^0 - \angle A$$

On dividing both sides by 2, we get:

$$\frac{\angle B + \angle C}{2} = \frac{180^0 - \angle A}{2}$$
$$\frac{\angle B + \angle C}{2} = 90^0 - \frac{\angle A}{2}$$

Applying sine angles on both the sides:

$$\sin\left(\frac{B+C}{2}\right) = \sin\left(90^0 - \frac{A}{2}\right)$$

Since

$$\sin\left(90^0 - 8\right) = \cos 8$$
$$\therefore \sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$$

Q7. Express $\sin 67^0 + \cos 75^0$ in terms of trigonometric ratios of angles between 0^0 and 45^0.

Difficulty level: Medium

Reasoning:

$$\cos\left(90^0 - \theta\right) = \sin\theta$$

Solution:

Given that: $\sin 67^0 + \cos 75^0 \ldots.(i)$

Since $\cos\left(90^0 - \theta\right) = \sin\theta$

By using property in equation (i) we get:

$$= \sin\left(90^0 - 23^0\right) + \cos\left(90^0 - 15^0\right)$$
$$= \cos 23^0 + \sin 15^0$$

Hence, the expression $\cos 23^0 + \sin 15^0$ has trigonometric ratios of angles between 0^0 and 45^0.

Chapter 8: Introduction to Trigonometry

Exercise 8.4 (Page 193 of Grade 10 NCERT Textbook)

Q1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Difficulty level: Medium

Reasoning:

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sec^2 A = 1 + \tan^2 A$$

Solution:

Consider a ΔABC with $\angle B = 90°$

Using the Trigonometric Identity,

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1+\cot^2 A} \quad \text{(By taking reciprocal both the sides)}$$
$$\sin^2 A = \frac{1}{1+\cot^2 A} \quad \left(\text{As } \frac{1}{\operatorname{cosec}^2 A} = \sin^2 A\right)$$

Therefore,

$$\sin A = \pm \frac{1}{\sqrt{1+\cot^2 A}}$$

For any sine value with respect to an angle in a triangle, sine value will never be negative. Since, sine value will be negative for all angles greater than 180°.

Therefore, $\sin A = \dfrac{1}{\sqrt{1+\cot^2 A}}$

We know that, $\tan A = \dfrac{\sin A}{\cos A}$

However, Trigonometric Function, $\cot A = \dfrac{\cos A}{\operatorname{Sin} A}$

Therefore, Trigonometric Function, $\tan A = \dfrac{1}{\cot A}$

Also, $\sec^2 A = 1 + \tan^2 A$ (Trigonometric Identity)

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Q2. Write all the other trigonometric ratios of $\angle A$ in terms of sec A.

Difficulty level: Medium

Reasoning:

$$\sin^2 A + \cos^2 A = 1$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sec^2 A = 1 + \tan^2 A$$

Solution:

We know that,

Trigonometric Function, $\cos A = \dfrac{1}{\sec A}$ $\quad \ldots$Equation (1)

Also,

$\sin^2 A + \cos^2 A = 1$ (Trigonometric identity)

$\sin^2 A = 1 - \cos^2 A$ (By transposing)

Using value of cos A from Equation (1) and simplifying further,

$$\sin A = \sqrt{1 - \left(\frac{1}{\sec A}\right)^2}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}$$

$$= \frac{\sqrt{\sec^2 A - 1}}{\sec A} \qquad \ldots\text{Equation (2)}$$

$\tan^2 A + 1 = \sec^2 A$ (Trigonometric identity)

$\tan^2 A = \sec^2 A - 1$ (By transposing)

Trigonometric Function,

$$\tan A = \sqrt{\sec^2 A - 1} \qquad \text{... Equation } (3)$$

$$\cot A = \frac{\cos A}{\sin A}$$

$$= \frac{\dfrac{1}{\sec A}}{\dfrac{\sqrt{\sec^2 A - 1}}{\sec A}} \qquad \ldots\left(\text{By substituting Equations } (1) \text{ and } (2)\right)$$

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A}$$

$$= \frac{\sec A}{\sqrt{\sec^2 A - 1}} \qquad \left(\text{By substituting Equation } (2) \text{ and simplifying}\right)$$

Q3. Evaluate

(i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

Difficulty level: Medium

Reasoning:

$\sin^2 A + \cos^2 A = 1$

$\sin(90^\circ - \theta) = \cos\theta$

$\cos(90^\circ - \theta) = \sin\theta$

Solution:

(i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

$$= \frac{\left[\sin\left(90^\circ - 27\right)\right]^2 + \sin^2 27}{\left[\cos\left(90^\circ - 73^\circ\right)\right]^2 + \cos^2 73^\circ}$$

$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ} \qquad \left(\sin(90^\circ - \theta) = \cos\theta \;\&\; \cos(90^\circ - \theta) = \sin\theta\right)$$

$$= \frac{1}{1} \qquad \left(\text{By Identity } \sin^2 A + \cos^2 A = 1\right)$$

$$= 1$$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$= \sin 25^\circ \left[\cos\left(90^\circ - 25^\circ\right)\right] + \cos 25^\circ \left[\sin\left(90^\circ - 25^\circ\right)\right]$

$= \sin 25^\circ . \sin 25^\circ + \cos 25^\circ . \cos 25^\circ \quad \left[\because \sin\left(90^\circ - \theta\right) = \cos\theta \ \& \ \cos\left(90^\circ - \theta\right) = \sin\theta\right]$

$= \sin^2 25^\circ + \cos^2 25^\circ$

$= 1 \quad \left(\text{By Identity } \sin^2 A + \cos^2 A = 1\right)$

Q4. Choose the correct option. Justify your choice.

(i) $9 \sec 2A - 9 \tan 2A =$ __________

(A) 1

(B) 9

(C) 8

(D) 0

(ii) $(1 + \tan\theta + \sec\theta)\ (1 + \cot\theta - \operatorname{cosec}\theta)$

(A) 0

(B) 1

(C) 2

(D) -1

(iii) $(\sec A + \tan A)\ (1 - \sin A) =$ __________

(A) $\sec A$

(B) $\sin A$

(C) $\operatorname{cosec} A$

(D) $\cos A$

(iv) $\dfrac{1+\tan^2 A}{1+\cot^2 A}$

(A) sec2A

(B) -1

(C) cot2A

(D) tan2A

Difficulty level: Medium

Reasoning:

$\sin^2 A + \cos^2 A = 1$

$\text{cosec}^2 A = 1 + \cot^2 A$

$\sec^2 A = 1 + \tan^2 A$

Solution:

(i) $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1$ [By the identity, $1 + \sec^2 A = \tan^2 A$, Hence $\sec^2 A - \tan^2 A = 1$]

$= 9$

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta)$ ……..Equation (1)

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

And

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\text{cosec}(x) = \frac{1}{\sin(x)}$$

By substituting the above function in Equation (1),

$$\Rightarrow \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$$ (By taking LCM and multiplying)

$$= \frac{(\sin\theta + \cos\theta)^2 - (1)^2}{\sin\theta\cos\theta}$$ (Using $a^2 - b^2 = (a+b)(a-b)$)

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$ (Using identify $\sin^2\theta + \cos^2\theta = 1$)

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$$

Hence, option(C)iscorrect.

(iii) $(\sec A + \tan A)(1 - \sin A)$ ......(1)

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

And

$$\sec(x) = \frac{1}{\cos(x)}$$

By substituting the above function in Equation (1),

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A} \quad (\text{By identify } \sin^2\theta + \cos^2\theta = 1, \text{Hence } 1 - \sin^2\theta = \cos^2\theta)$$

$$= \cos A$$

Hence, option (D) is correct.

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A}$

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$= \frac{1}{\tan(x)}$$

By substituting the above function in Equation (1),

$$\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{1+\dfrac{\sin^2 A}{\cos^2 A}}{1+\dfrac{\cos^2 A}{\sin^2 A}}$$

$$=\frac{\dfrac{\cos^2 A+\sin^2 A}{\cancel{\cos^2 A}}}{\dfrac{\sin^2 A+\cos^2 A}{\sin^2 A}}$$

$$=\frac{\dfrac{1}{\cancel{\cos^2 A}}}{\dfrac{1}{\sin^2 A}}$$

$$=\frac{\sin^2 A}{\cos^2 A}$$

$$=\tan^2 A$$

Hence, option (D)is correct.

Q5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\operatorname{cosec}\theta-\cot\theta)^2=\dfrac{1-\cos\theta}{1+\cos\theta}$

(ii) $\dfrac{\cos A}{1+\sin A}+\dfrac{1+\sin A}{\cos A}=2\sec A$

(iii) $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$

(iv) $\dfrac{1+\sec A}{\sec A}=\dfrac{\sin^2 A}{1-\cos A}$

(v) $\dfrac{\cos A-\sin A+1}{\cos A+\sin A+1}=\operatorname{cosec} A+\cot A$

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\dfrac{\sin\theta-2\sin^3\theta}{2\cos\theta-\cos\theta}=\tan\theta$

(viii) $(\sin A+\operatorname{cosec} A)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$

(ix) $(\operatorname{cosec} A-\sin A)(\sec A-\cos A)=\dfrac{1}{\tan A+\cot A}$

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

Difficulty level: Medium

Reasoning:

$$\sin^2 A + \cos^2 A = 1$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sec^2 A = 1 + \tan^2 A$$

Solution:

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

L.H.S=$(\operatorname{cosec}\theta - \cot\theta)^2$ ……..(1)

We know that the trigonometric functions,

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

By substituting the above function in Equation (1)

$$(\operatorname{cosec}\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{(\sin\theta)^2}$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} \quad \left(\text{By Identity } \sin^2 A+\cos^2 A = 1 \text{ Hence, } 1-\cos^2 A=\sin^2 A\right)$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} \quad \left[\text{Using } a^2-b^2=(a+b)(a-b)\right]$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

=RHS

(ii) $\dfrac{\cos A}{1+\sin A}+\dfrac{1+\sin A}{\cos A}=2\sec A$

$$\text{L.H.S}=\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$$
$$=\frac{\cos^2A+(1+\sin A)^2}{(1+\sin A)(\cos A)}$$
$$=\frac{\cos^2A+1+\sin^2A+2\sin A}{(1+\sin A)(\cos A)}$$
$$=\frac{\sin^2A+\cos^2A+1+2\sin A}{(1+\sin A)(\cos A)}$$
$$=\frac{1+1+2\sin A}{(1+\sin A)(\cos A)} \qquad \left(\text{By identify } \sin^2A+\cos^2A=1\right)$$
$$=\frac{2+2\sin A}{(1+\sin A)(\cos A)}$$
$$=\frac{2(1+\sin A)}{(1+\sin A)(\cos A)}$$
$$=\frac{2}{\cos A}$$
$$=2\sec A$$
$$=\text{R.H.S}$$

(iii) $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\sec\theta\ \operatorname{cosec}\theta$

$$\text{LHS}=\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta} \qquad \ldots\ldots(1)$$

We know that the trigonometric functions,

$$\tan(x)=\frac{\sin(x)}{\cos(x)}$$
$$\cot(x)=\frac{\cos(x)}{\sin(x)}=\frac{1}{\tan(x)}$$

By substituting the above relations in Equation (1),

$$=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\cos\theta}{\sin\theta}}$$

$$=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta}-\frac{\cos^2\theta}{\sin\theta}\right]$$

$$=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

Using $a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$

$$=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{(\sin\theta-\cos\theta)\left(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta\right)}{\sin\theta\cos\theta}\right]$$

$$=\frac{(1+\sin\theta\cos\theta)}{(\sin\theta\cos\theta)}$$ (By Identity $\sin^2A+\cos^2A=1$)

$$=\frac{1}{\sin\theta\cos\theta}+\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$=1+\sec\theta\operatorname{cosec}\theta$

=R.H.S.

(iv) $\dfrac{1+\sec A}{\sec A}=\dfrac{\sin^2A}{1-\cos A}$

L.H.S=$\dfrac{1+\sec A}{\sec A}$ ……(1)

We know that the trigonometric functions,

$$\sec(x)=\frac{1}{\cos(x)}$$

By substituting the above function in Equation (1),

$$\frac{1+\sec A}{\sec A}=\frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$=\frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$$

$$=\frac{\cos A+1}{\cos A}\times\frac{\cos A}{1}$$

$$=(1+\cos A)$$

By multiplying $(1-\cos A)$, in both denominator and numerator

$$\Rightarrow\frac{(1-\cos A)(1+\cos A)}{(1-\cos A)}$$

$$=\frac{1-\cos^2 A}{1-\cos A}$$

$$=\frac{\sin^2 A}{1-\cos A}\quad\left[\text{By Identity } \sin^2 A+\cos^2 A=1\right]$$

$$=\text{R. H.S}$$

(v) $\dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec} A+\cot A$

$$\text{L.H.S}=\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$$

Diving both numerator and denominator by $\sin A$

$$\Rightarrow\frac{\frac{\cos A}{\sin A}-\frac{\sin A}{\sin A}+\frac{1}{\sin A}}{\frac{\cos A}{\sin A}+\frac{\sin A}{\sin A}-\frac{1}{\sin A}}$$

We know that the trigonometric functions,

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

Weget

$$\Rightarrow \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$\Rightarrow \frac{\cot A - (1 - \operatorname{cosec} A)}{\cot A + (1 - \operatorname{cosec} A)}$$

We know that, $1 + \cot^2 A = \operatorname{Cosec}^2 A$

Hence multiplying [cot A – (1 – cosec A)] in numerator and denominator

$$\Rightarrow \frac{[(\cot A) - (1 - \operatorname{cosec} A)][(\cot A) - (1 - \operatorname{cosec} A)]}{[(\cot A) + (1 - \operatorname{cosec} A)][(\cot A) - (1 - \operatorname{cosec} A)]}$$

$$= \frac{\left[\cot A - (1 - \operatorname{cosec} A)\right]^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2}$$

$$= \frac{\cot^2 A + (1 - \operatorname{cosec} A)^2 - 2\cot A(1 - \operatorname{cosec} A)}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A - 2\cot A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \frac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$$

$$= \frac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{-1 - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)}$$

$$= \operatorname{cosec} A + \cot A$$

$$= \text{R.H.S}$$

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}}$ =sec A+tanA

$$\text{LHS}=\sqrt{\frac{1+\sin A}{1-\sin A}} \qquad \ldots\ldots \ (1)$$

Multiplying and dividing by $\sqrt{(1+\sin A)}$

$$\Rightarrow \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$=\sqrt{\frac{(1+\sin A)^2}{(1-\sin^2 A)}} \qquad \left[\because a^2-b^2=(a-b)(a+b),\right]$$

$$=\frac{(1+\sin A)}{\sqrt{1-\sin^2 A}}$$

$$=\frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$=\frac{1+\sin A}{\cos A}$$

$$=\frac{1}{\cos A}+\frac{\sin A}{\cos A}$$

$$=\sec A+\tan A$$

=R.H.S

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta} = \tan\theta$

$$\text{L.H.S}=\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

Taking Sin θ and Cos θ common in both numerator and denominator respectively.

$$\Rightarrow \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta-1\right)}$$

By Identity $\sin^2 A + \cos^2 A = 1$ hence, $\cos^2 A = 1 - \sin^2 A$ and substituting this in the above equation,

$$\Rightarrow \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left\{2\left(1-\sin^2\theta\right)-1\right\}}$$

$$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2-2\sin^2\theta-1\right)}$$

$$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(1-2\sin^2\theta\right)}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

$$= \text{RHS}$$

(viii) $(\sin A+\operatorname{cosec} A)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$

$$\text{L.H.S}=(\sin A+\operatorname{cosec}A)^2+(\cos A+\sec A)^2$$

By using $(a+b)^2=a^2+2ab+b^2$

$$\Rightarrow \sin^2A+\operatorname{cosec}^2A+2\sin A\ \operatorname{cosec}A+\cos^2A+\sec^2A+2\cos A\ \ \sec A$$

By rearranging and using $\sec A=\dfrac{1}{\cos A}$ and $\operatorname{cosec} A=\dfrac{1}{\sin A}$

$$\Rightarrow \left(\sin^2A+\cos^2A\right)+\left(\operatorname{cosec}^2A+\sec^2A\right)+2\sin A\left(\frac{1}{\sin A}\right)+2\cos A\left(\frac{1}{\cos A}\right)$$

Hence $\left(\sin^2A+\cos^2A\right)=1$, $\operatorname{cosec}^2A=\left(1+\cot^2A\right)$ and $\left(\sec^2A-\tan^2A\right)=1$

$$\Rightarrow 1+1+\cot^2 A+1+\tan^2 A+2+2$$

$$=7+\tan^2 A+\cot^2 A$$

$$=\text{R.H.S}$$

(ix) $(\operatorname{cosec} A-\sin A)(\sec A-\cos A)=\dfrac{1}{\tan A+\cot A}$

$$\text{L.H.S}=(\operatorname{cosec} A-\sin A)(\sec A-\cos A) \qquad \ldots\ldots\ldots (1)$$

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We know that the trigonometric functions,

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

By substituting the above relations in Equation (1)

$$\Rightarrow \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$$

$$= \frac{\cos^2 A \sin^2 A}{\sin A \cos A}$$

$$= \frac{\sin A \cos A}{1}$$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} \quad \left[\because \left(\sin^2 A + \cos^2 A\right) = 1\right]$$

$$= \frac{1}{\dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}} \quad \left[\text{Dividing numerator and denominator by } (\sin A \cos A)\right]$$

$$= \frac{1}{\dfrac{\sin^2 A}{\sin A \cos A} + \dfrac{\cos^2 A}{\sin A \cos A}}$$

$$= \frac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$$

$$= \frac{1}{\tan A + \cot A}$$

$$= \text{RHS}$$

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

Taking LHS, $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)$

$$=\frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$=\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$=\frac{1}{\cos^2 A}\times\frac{\sin^2 A}{1}$$

$$=\tan^2 A$$

$$=\text{RHS}$$

Taking, $\left(\dfrac{1-\tan A}{1-\cot A}\right)^2$

$$=\left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$=\left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2$$

$$=\left((1-\tan A)\times\frac{\tan A}{\tan A-1}\right)^2$$

$$=(-\tan A)^2$$

$$=\tan^2 A$$

$$=\text{RHS}$$

Hence, L.H.S = R.H.S.

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